

# Complex Networks

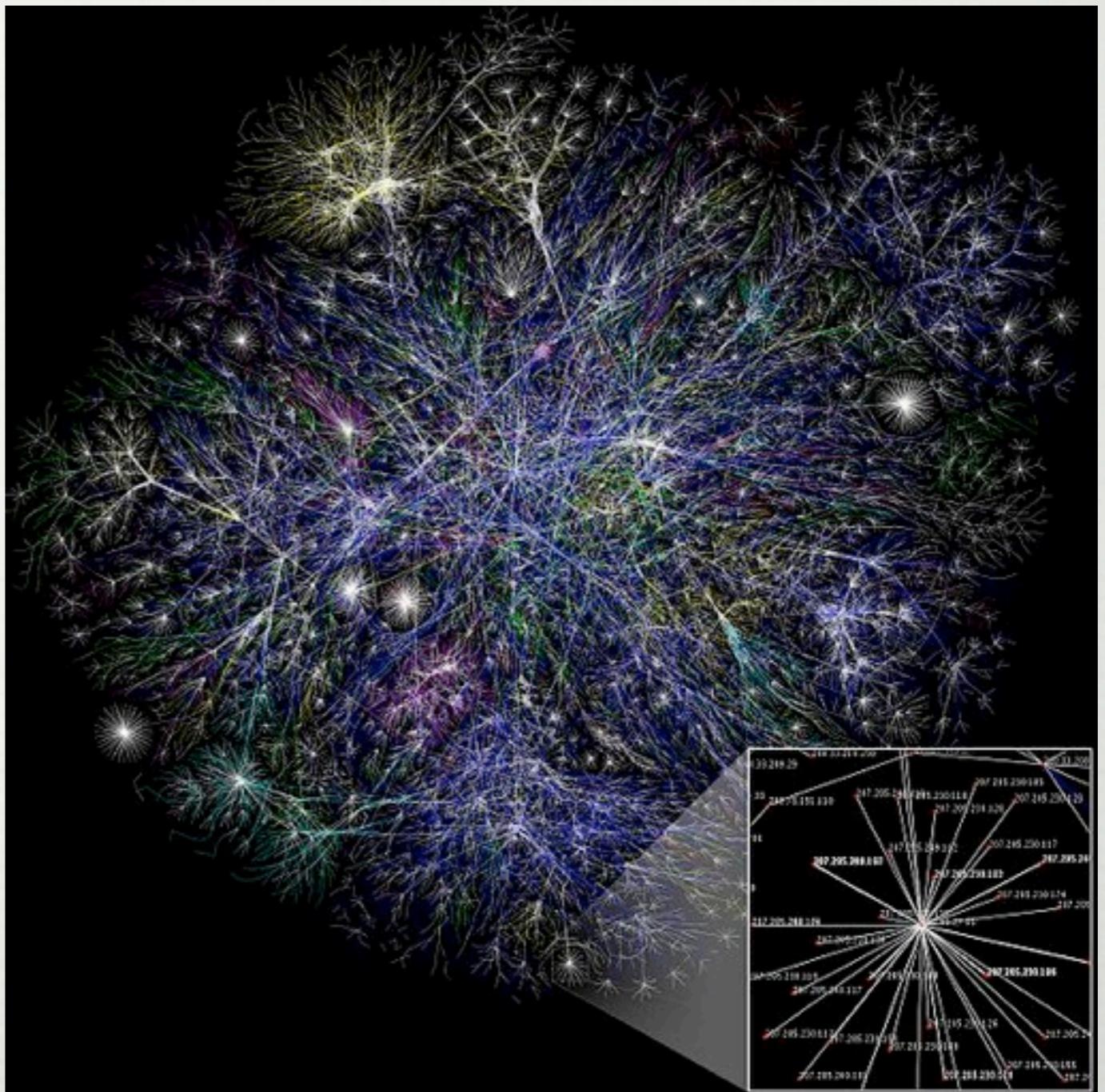
Теория сложных сетей

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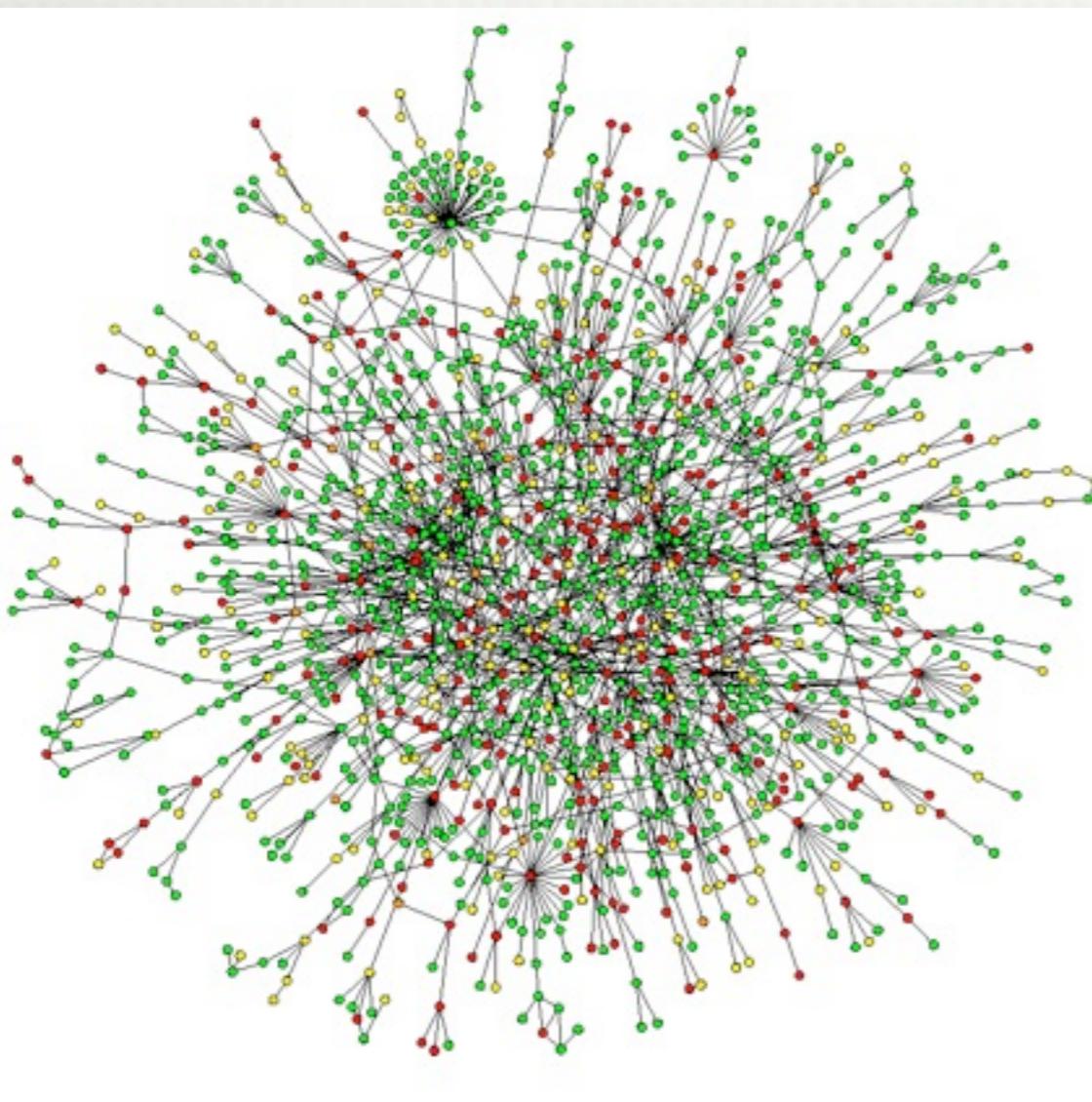
# КОМПЛЕКСНЫЕ СЕТИ

- СЕТЬ/ГРАФ: УЗЛЫ И СВЯЗИ МЕЖДУ НИМИ
- СЛОЖНЫЕ (COMPLEX NETWORKS) - НЕТРИВИАЛЬНАЯ ТОПОЛОГИЯ
- НЕ РЕГУЛЯРНЫЕ И НЕ СЛУЧАЙНЫЕ ПАТТЕРНЫ СОЕДИНЕНИЯ УЗЛОВ
- РАЗНООБРАЗНЫЕ
- УНИВЕРСАЛЬНЫЕ СВОЙСТВА

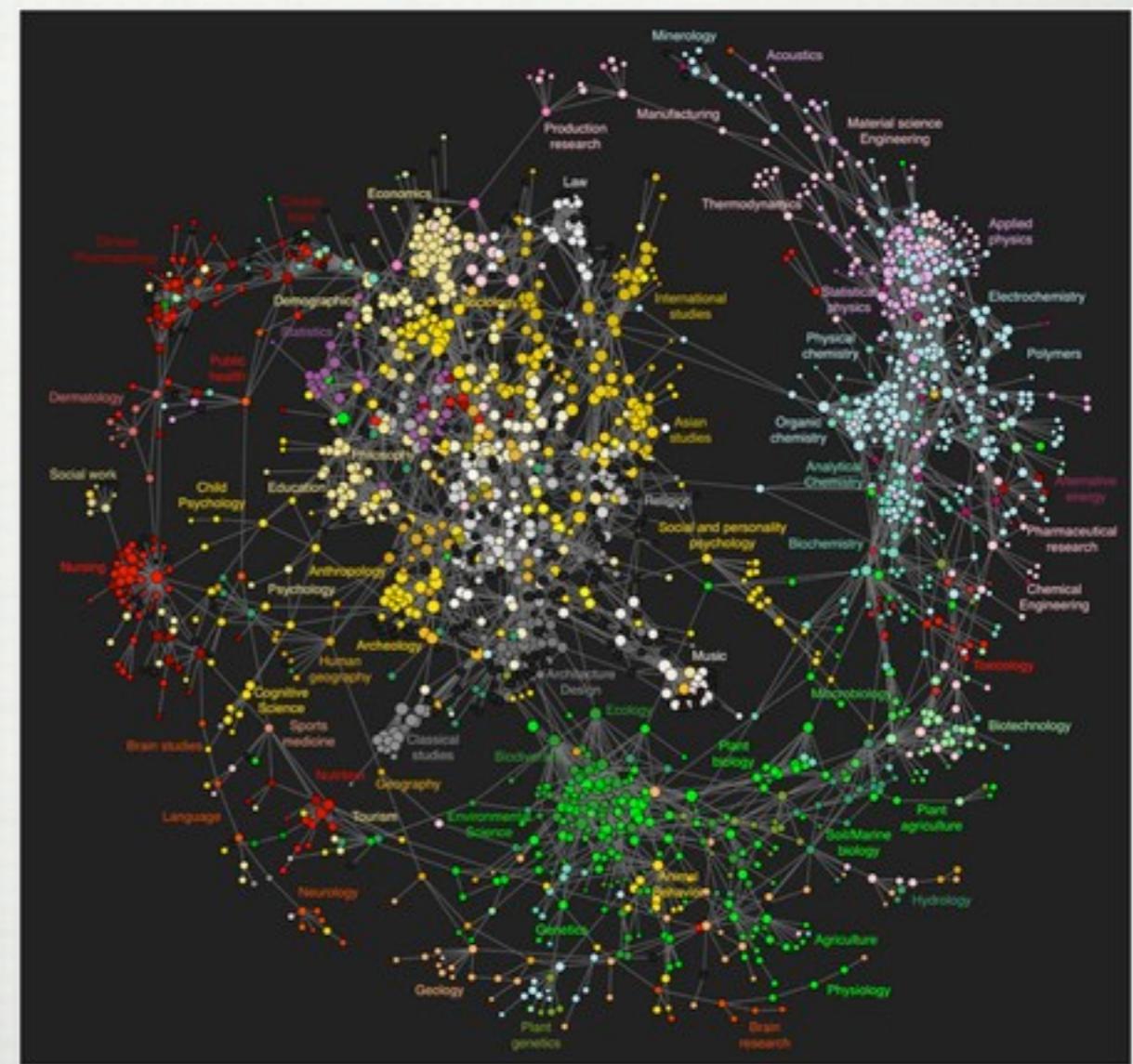


BARRETT LYON, OPTE.ORG. CLASS C NETWORKS

# КОМПЛЕКСНЫЕ СЕТИ

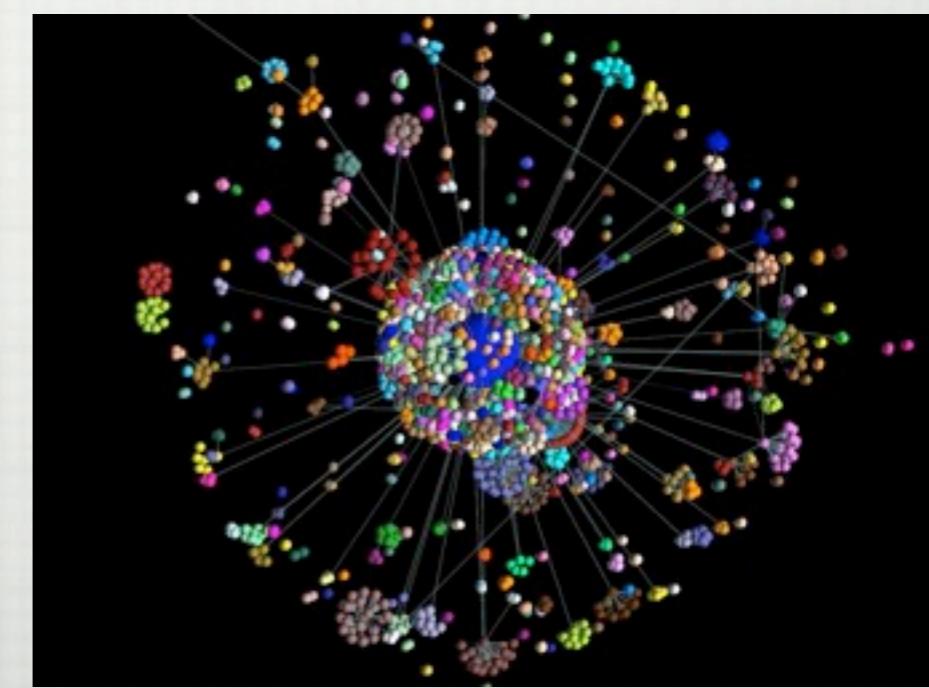
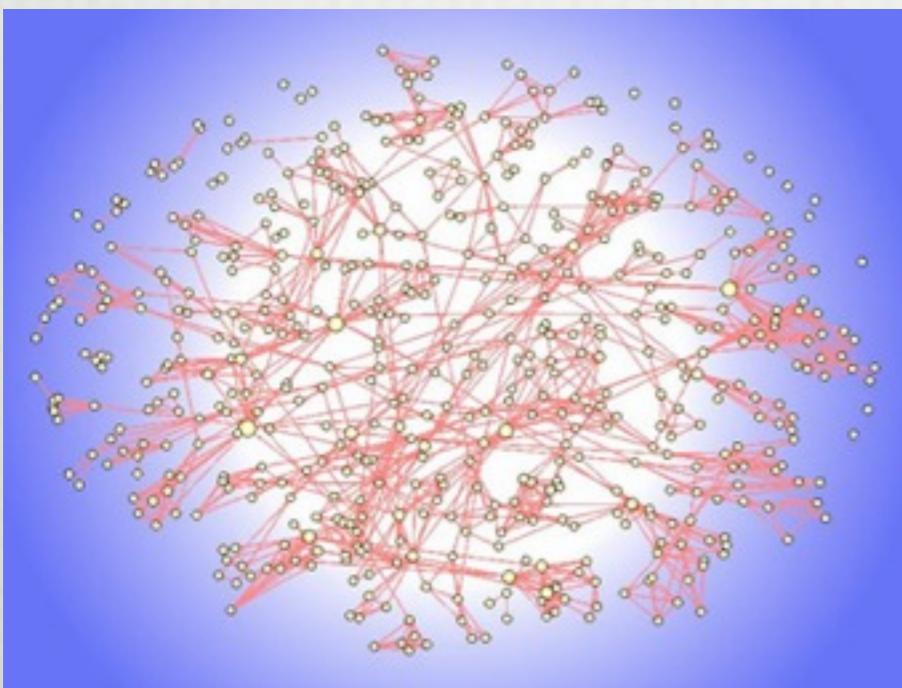
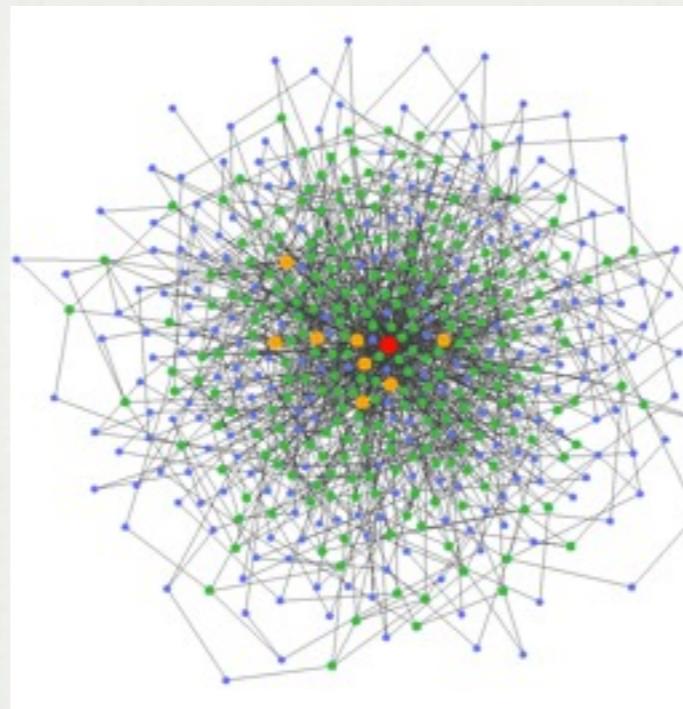


## PROTEIN INTERACTION

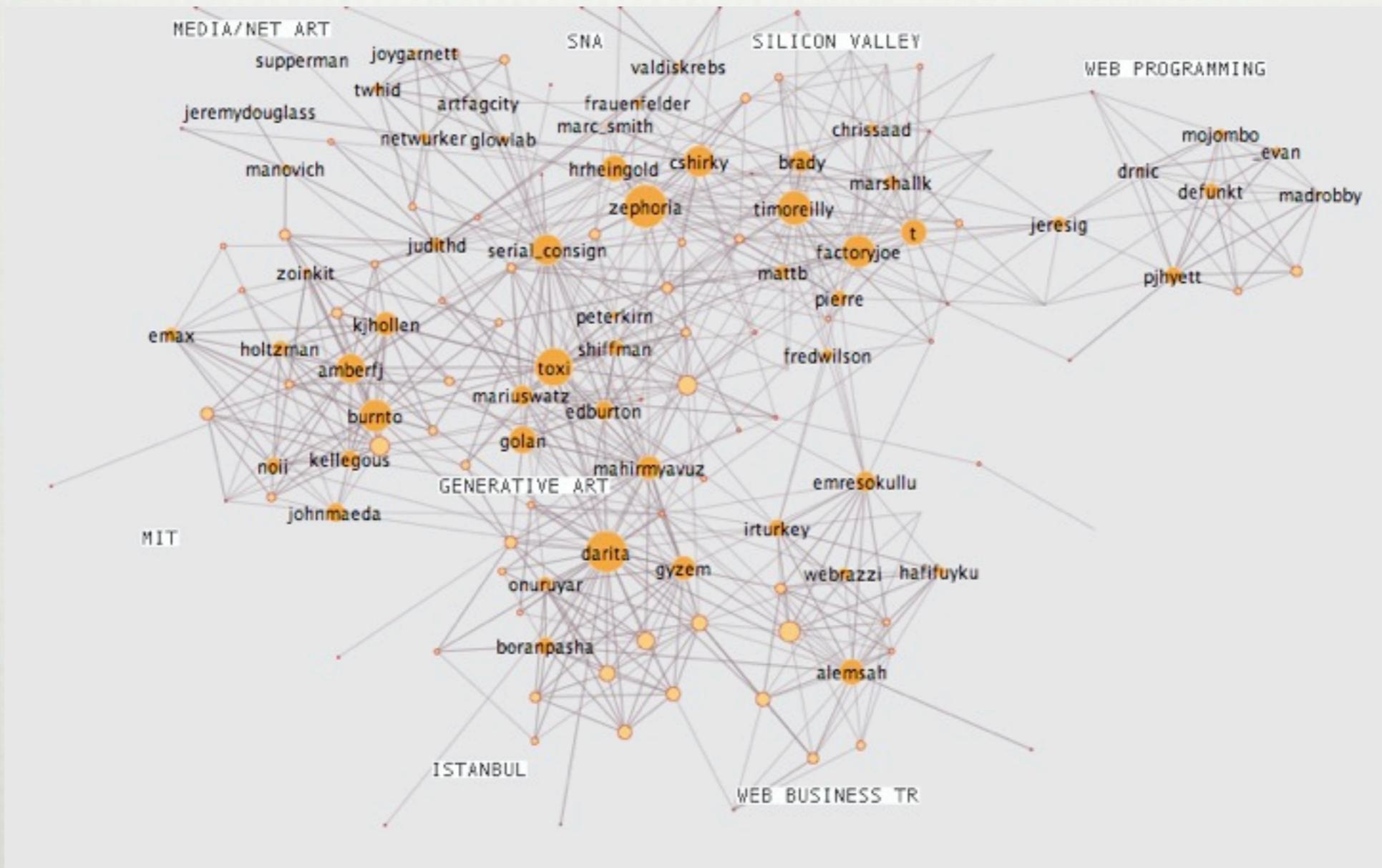


## LOG OF USERS SC.JOURNALS ACTIVITY

# КОМПЛЕКСНЫЕ СЕТИ



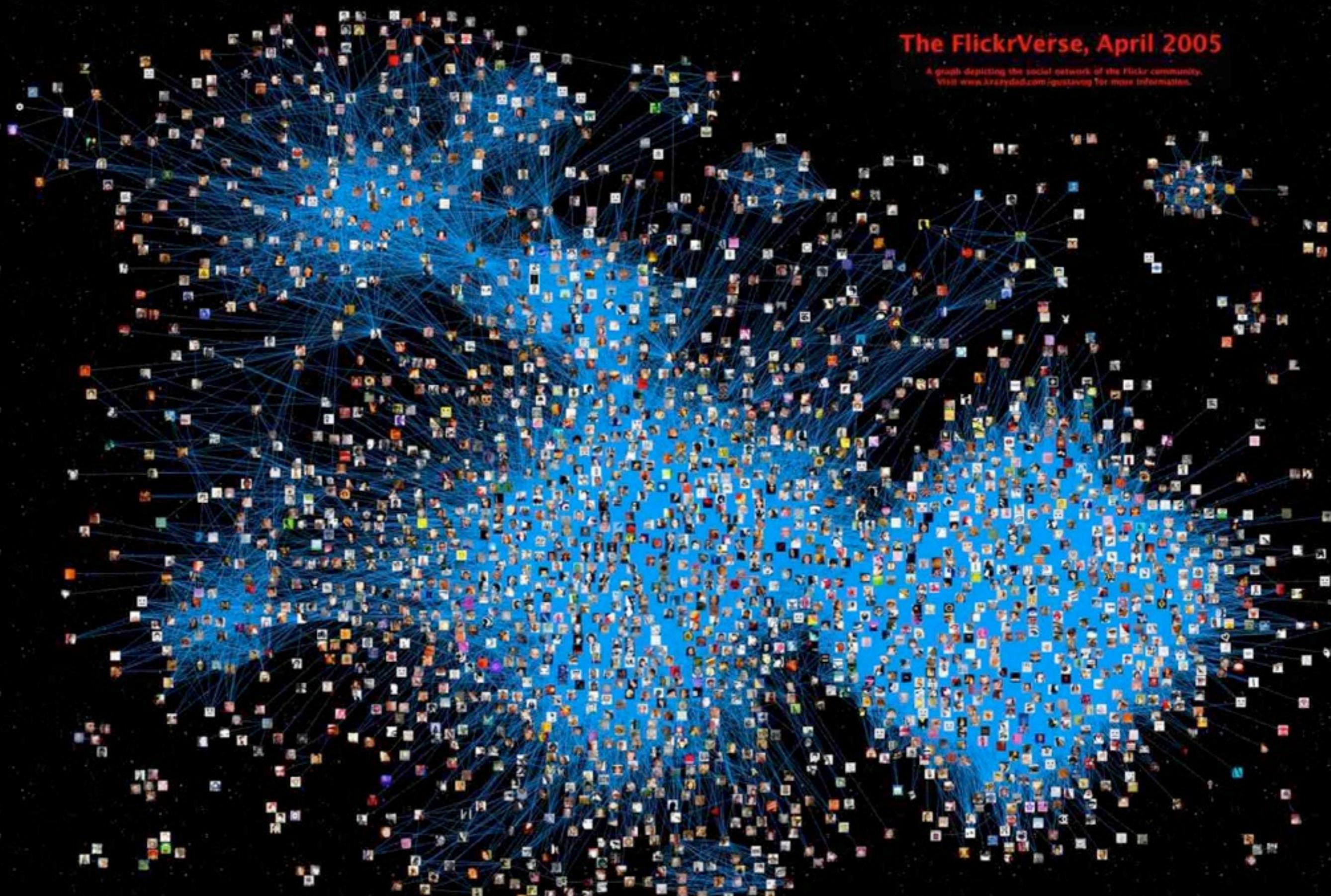
# КОМПЛЕКСНЫЕ СЕТИ



TWITTER FOLLOWERS

# The FlickrVerse, April 2005

A graph depicting the social network of the Flickr-community.  
Visit [www.krazydude.com/flickrverse](http://www.krazydude.com/flickrverse) for more information.



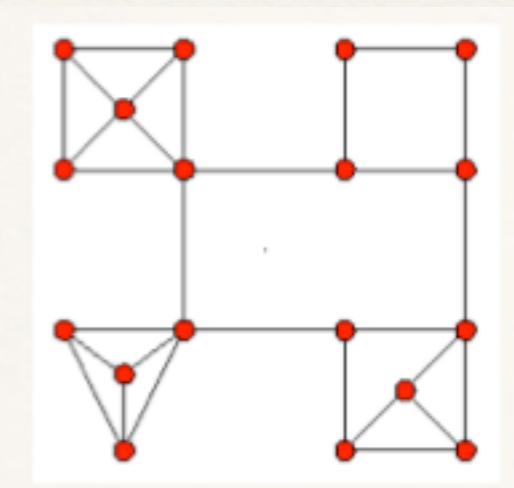
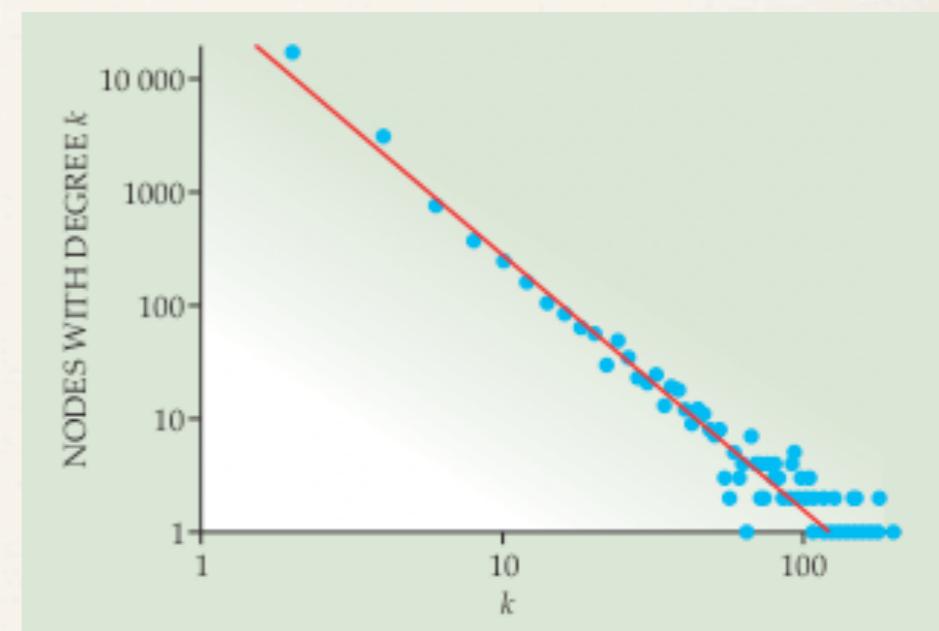
# Примеры сетей

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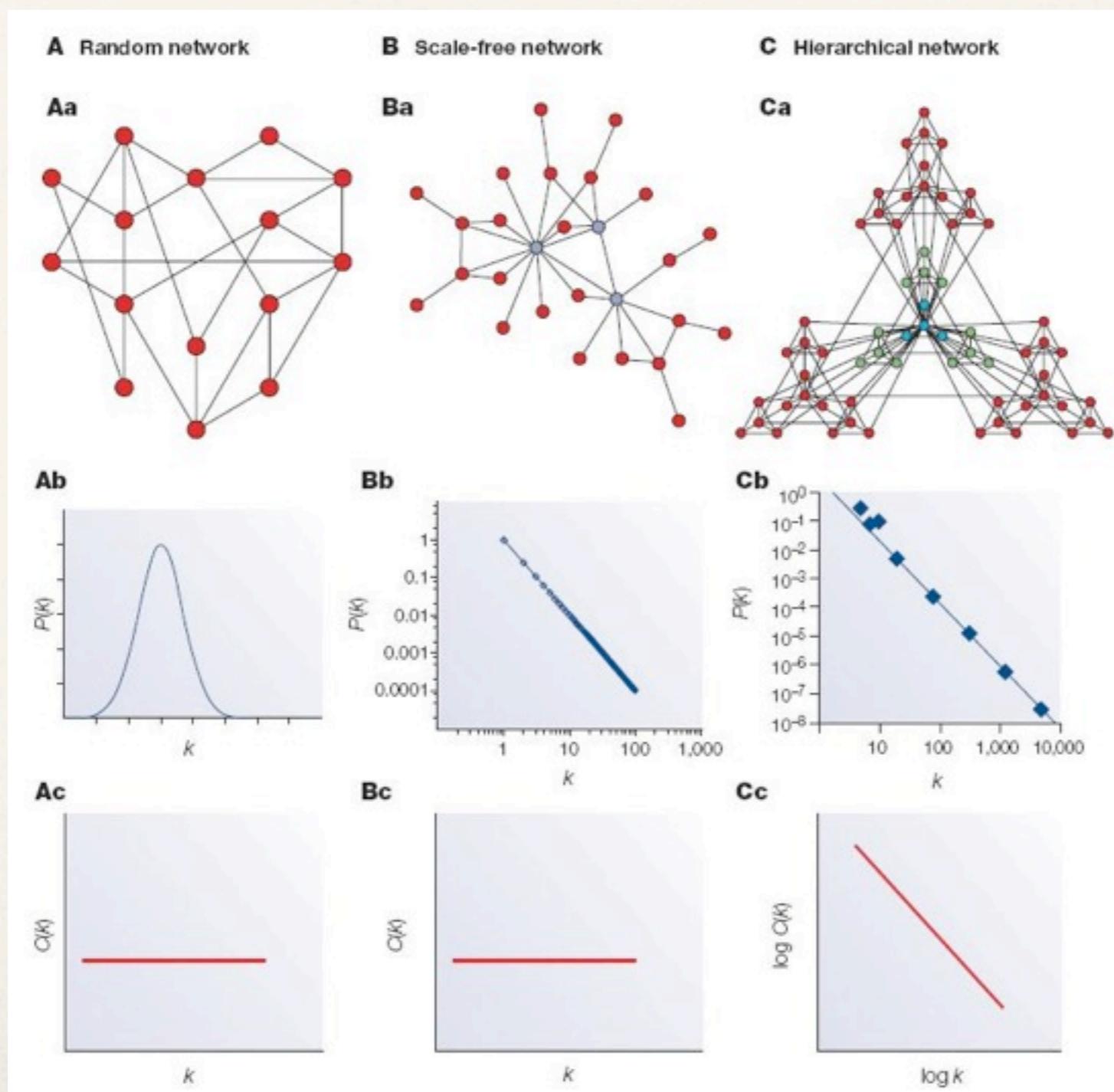
- ❖ Интернет и веб граф: связь между рутерами, веб ссылки
- ❖ Социальные сети: дружба, общения, сотрудничество, работа
- ❖ Биологические сети: взаимодействие белков, сети метаболизма
- ❖ Семантические сети: связь слов, понятий
- ❖ Коммуникационные сети: email, skype, phone calls
- ❖ Технологические сети: линии электро передач, транспортные сети
- ❖ Бизнес, экономические, торговые, политические сети
- ❖ Рекомендательные сервисы: сети музыки, фильмов, книг

# Свойства

- ❖ Статистические:
  - ❖ Функция распределения степеней узлов
  - ❖ Корреляционные функции
- ❖ Структурные:
  - ❖ Диаметр или характеристический размер
  - ❖ Степень кластеризации
  - ❖ Наличие гигантской связанной компоненты
  - ❖ Иерархическая структура сети

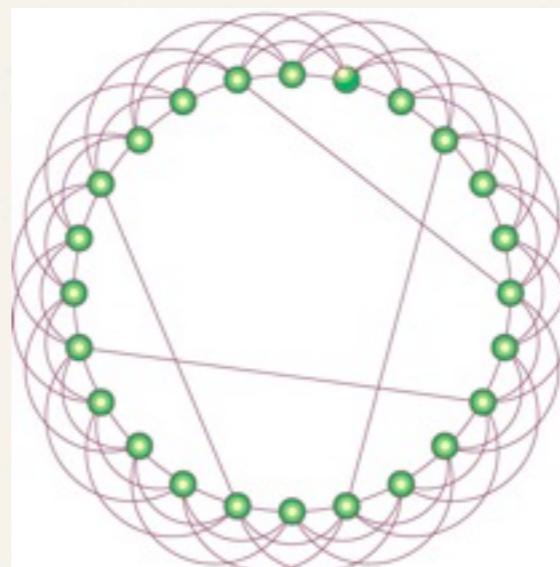


# Свойства

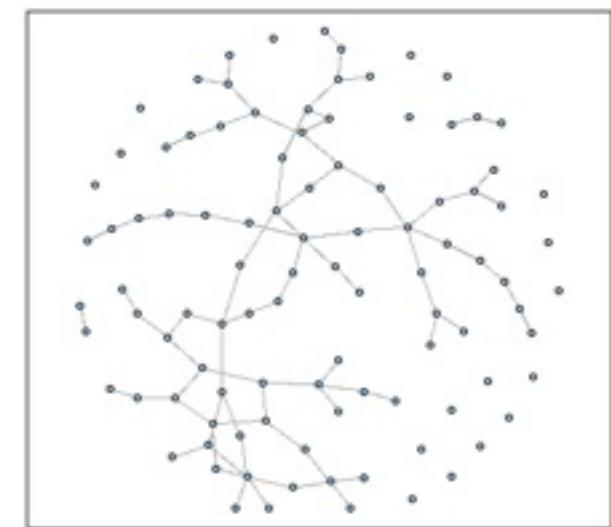


# Модели формирования сетей

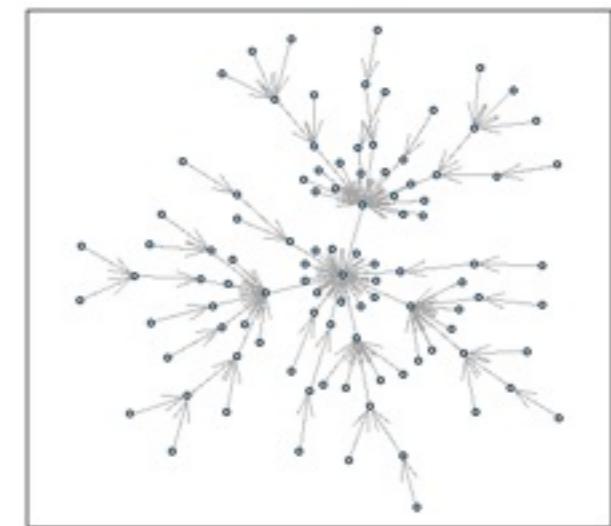
- ❖ Случайный граф (Erdos & Renyi, 1956)



- ❖ Модель “малого мира” (Watts & Strogatz, 1998)



- ❖ Модель предпочтительного присоединения (Barabasi & Albert, 1999 )



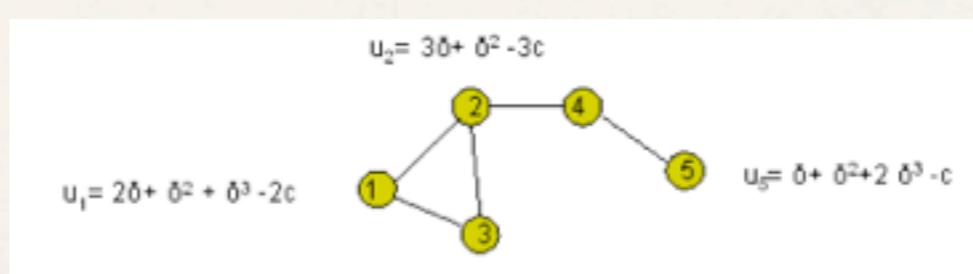
# Стратегические модели

- ❖ Утилитарные модели: функция полезности, стоимость связей
- ❖ Стабильность и эффективность сетей (Jackson & Wolinsky, 1996)

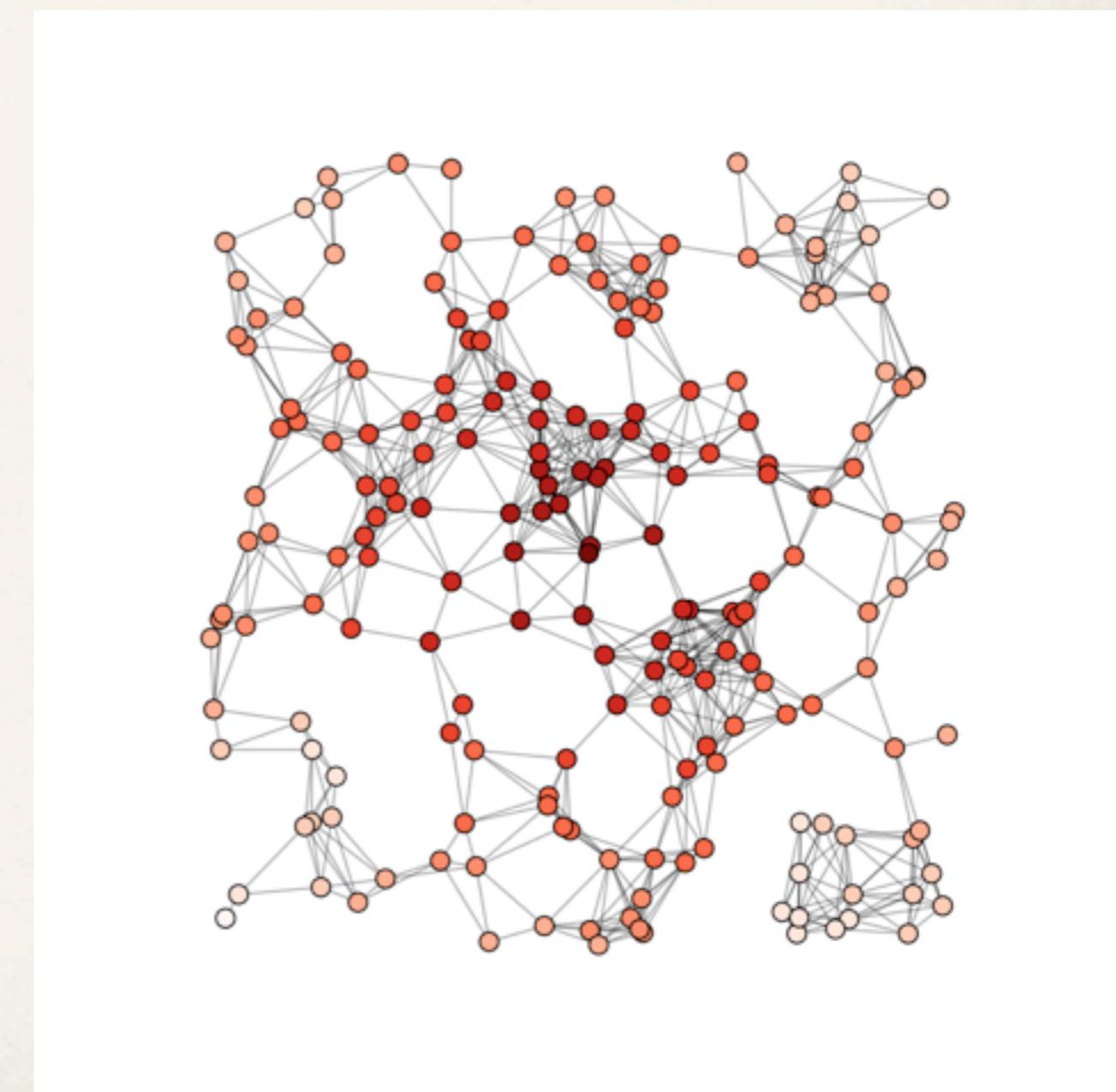
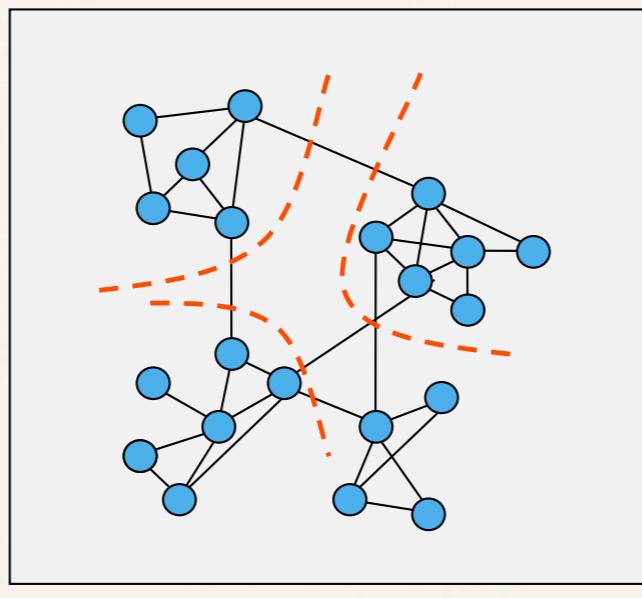
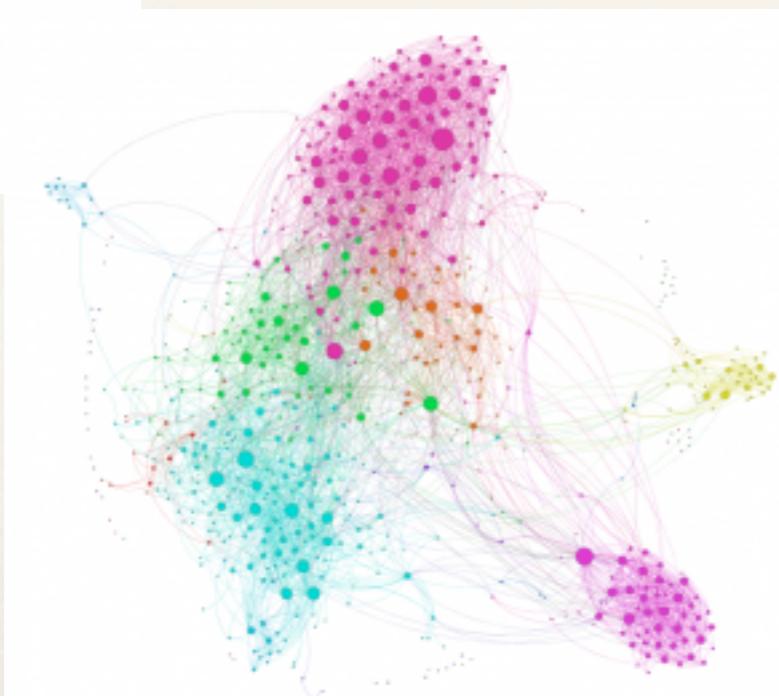
- ❖ Модель расстояний

$$u_i(g) = \sum_{j \neq i: i \text{ and } j \text{ are path-connected in } g} (\delta_{ij})^{p_{ij}(g)} - \sum_{j \neq i: ij \in g} c_{ij},$$

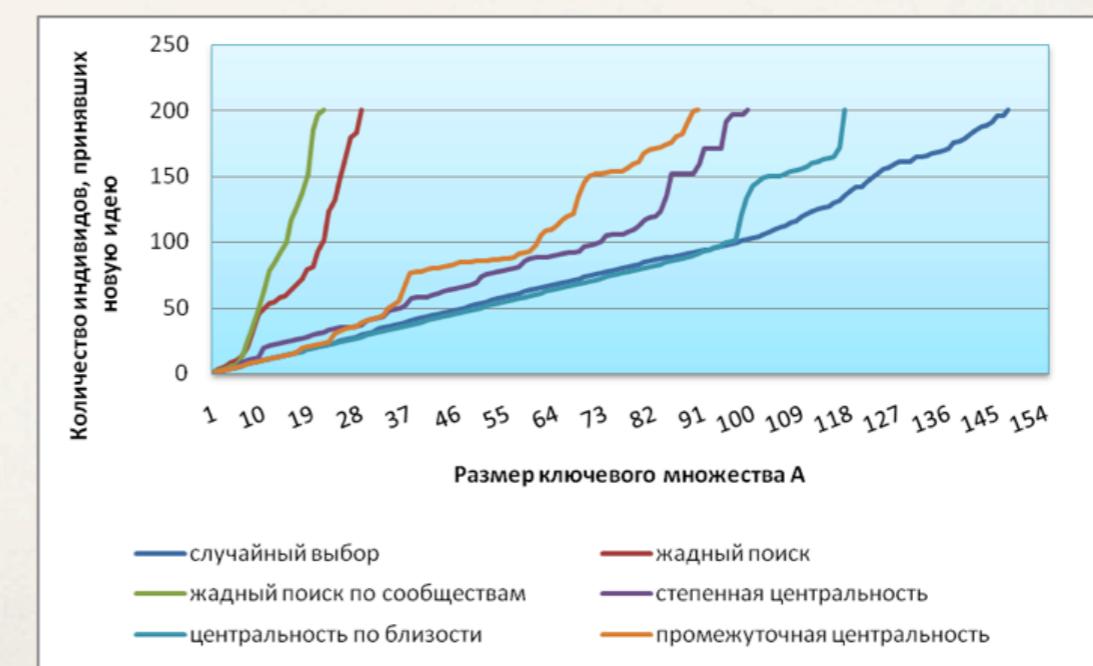
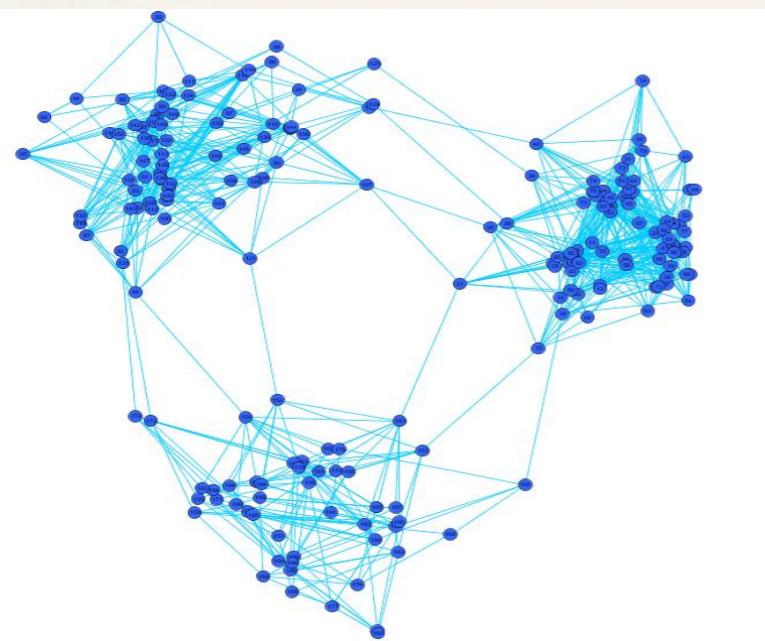
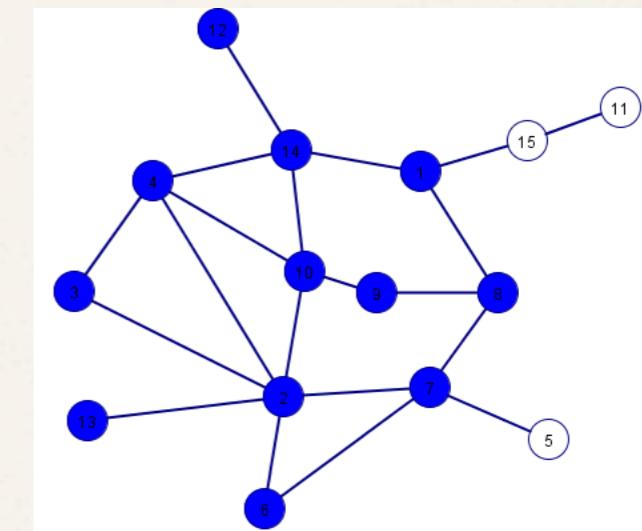
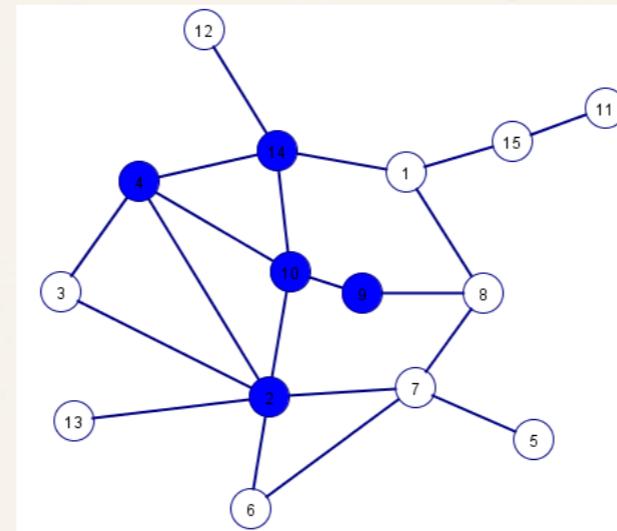
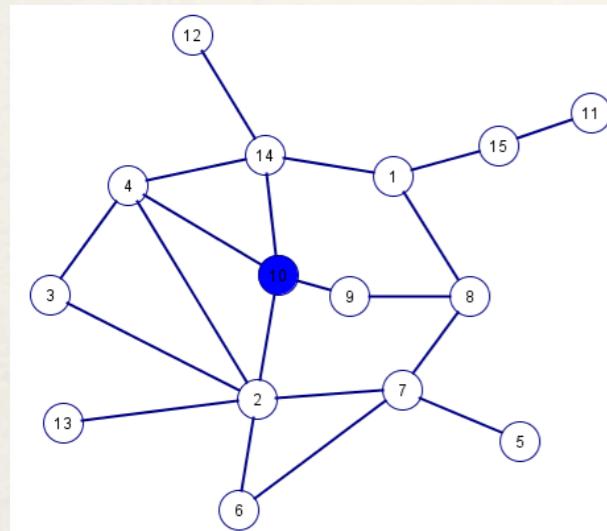
- ❖ Модель соавторов



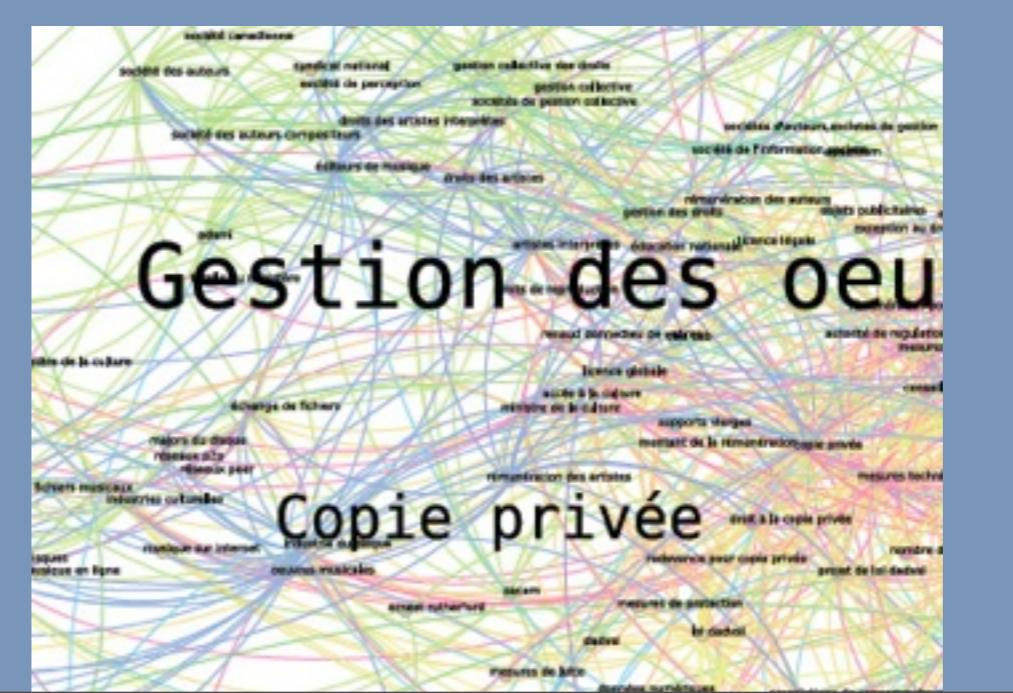
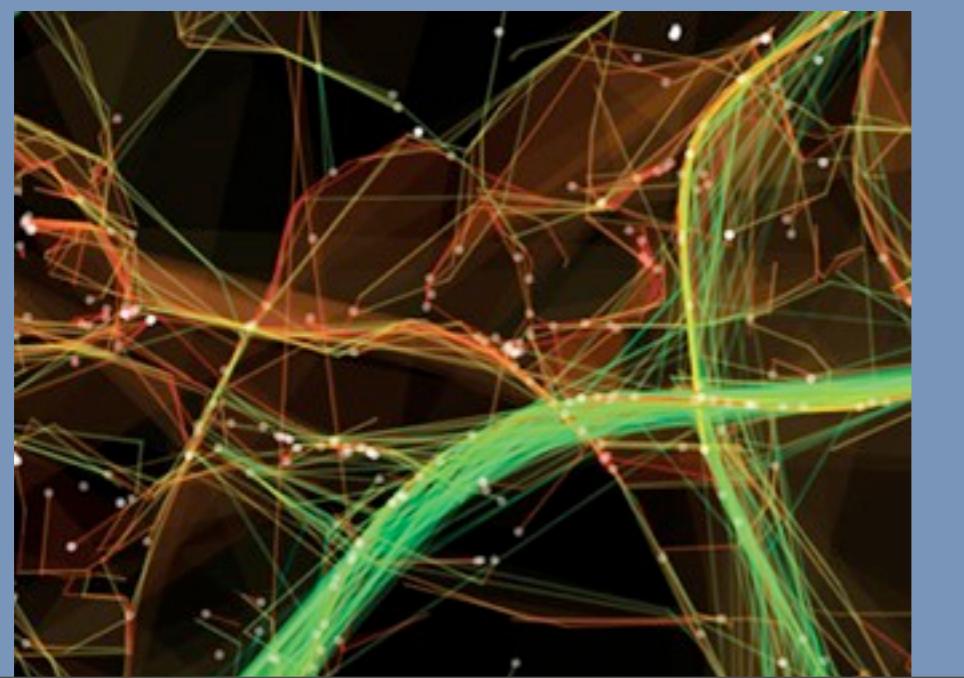
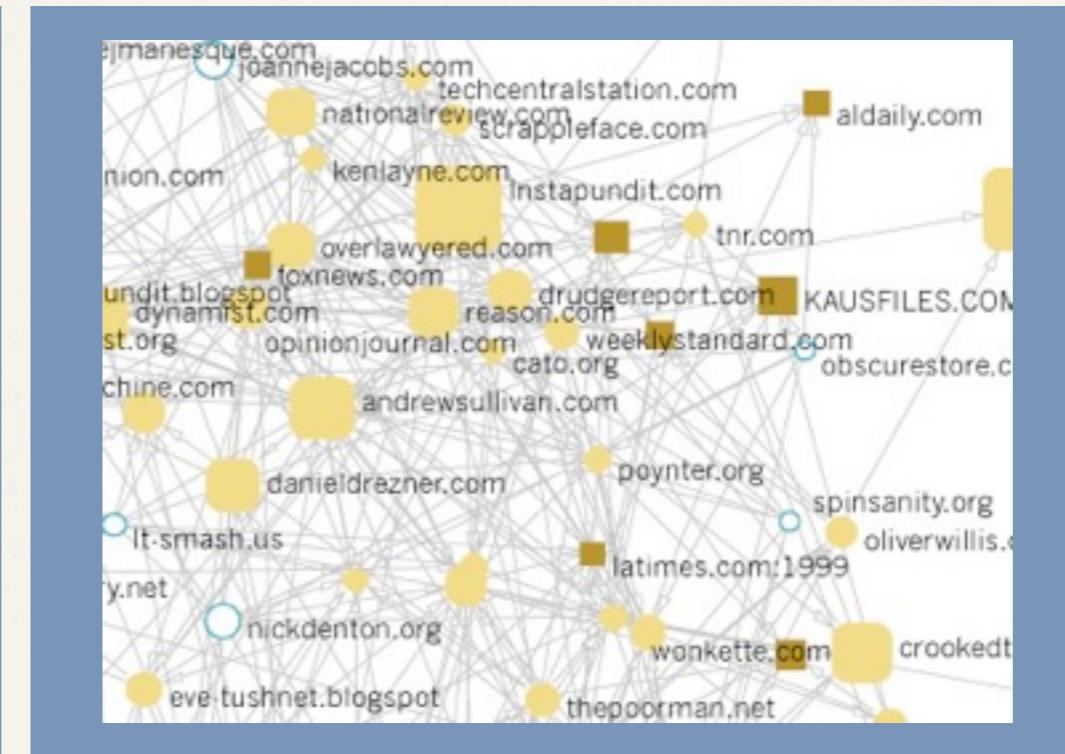
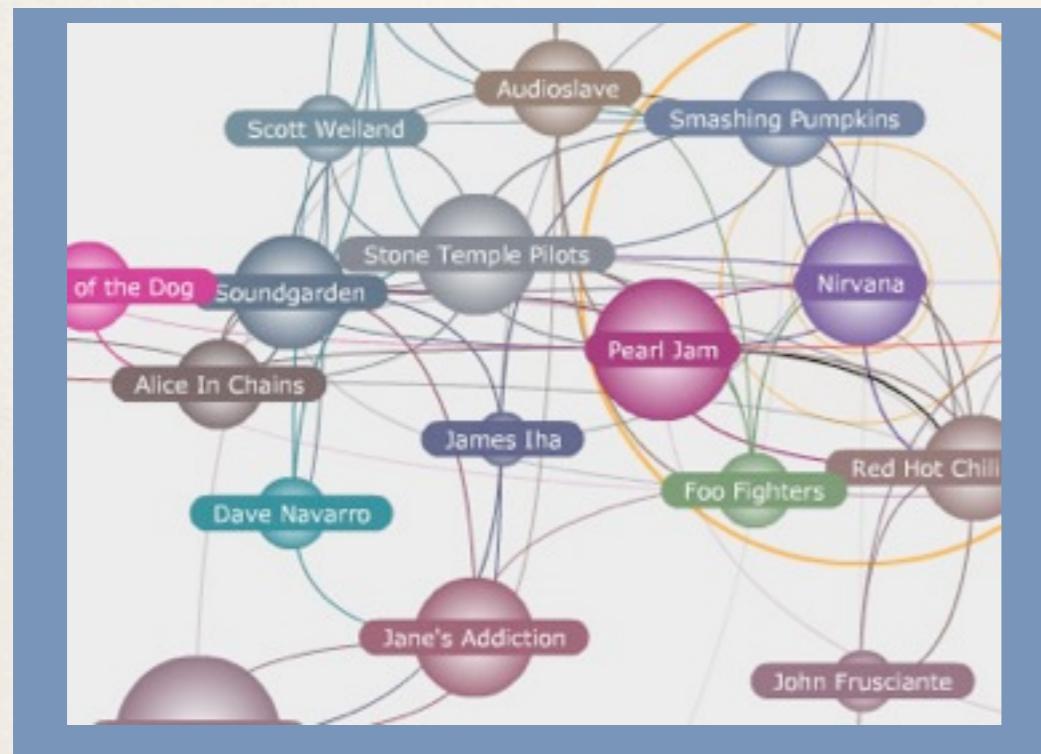
# Нахождение сообществ



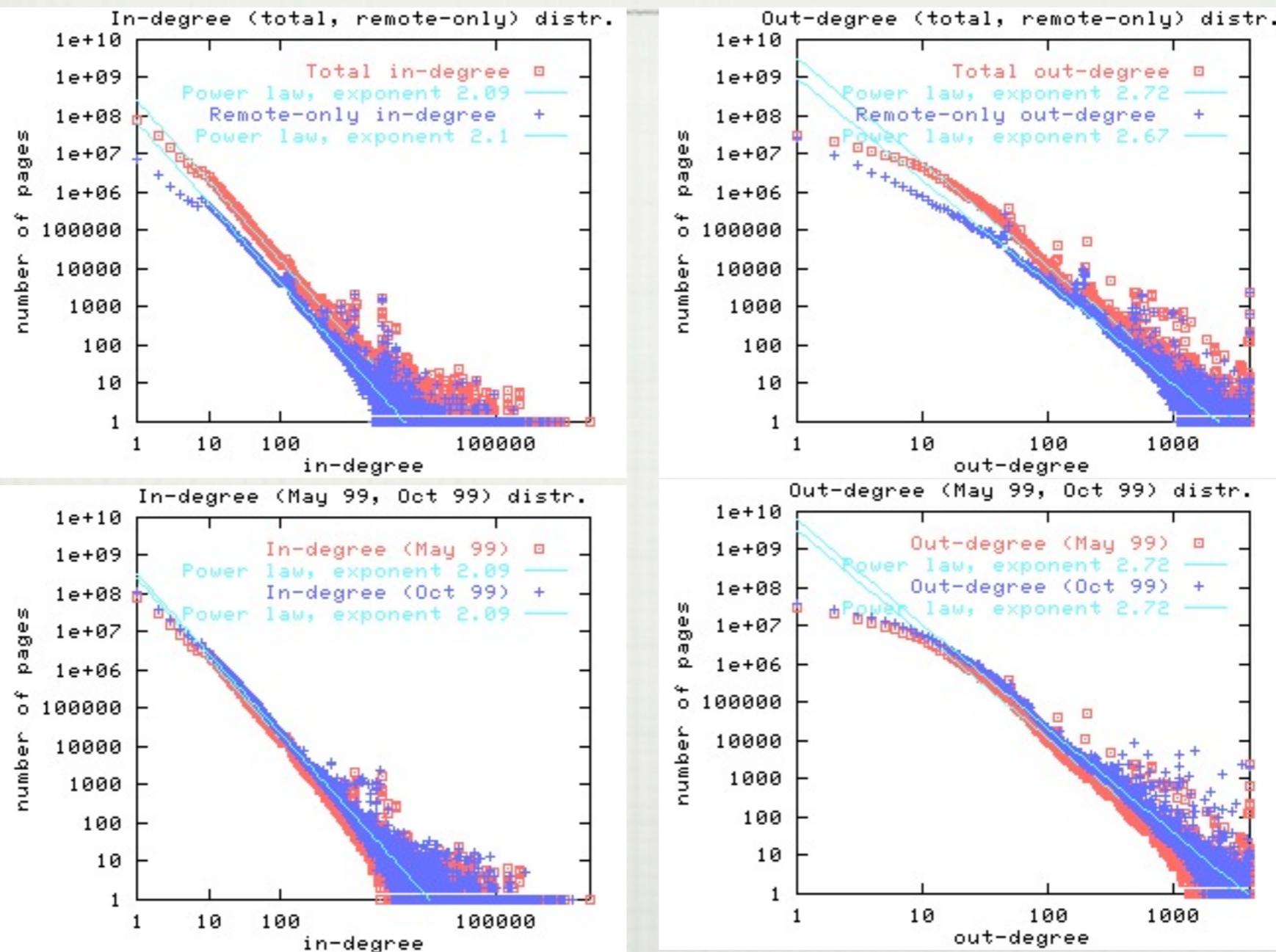
# Распространение информации



# Визуализация сетей

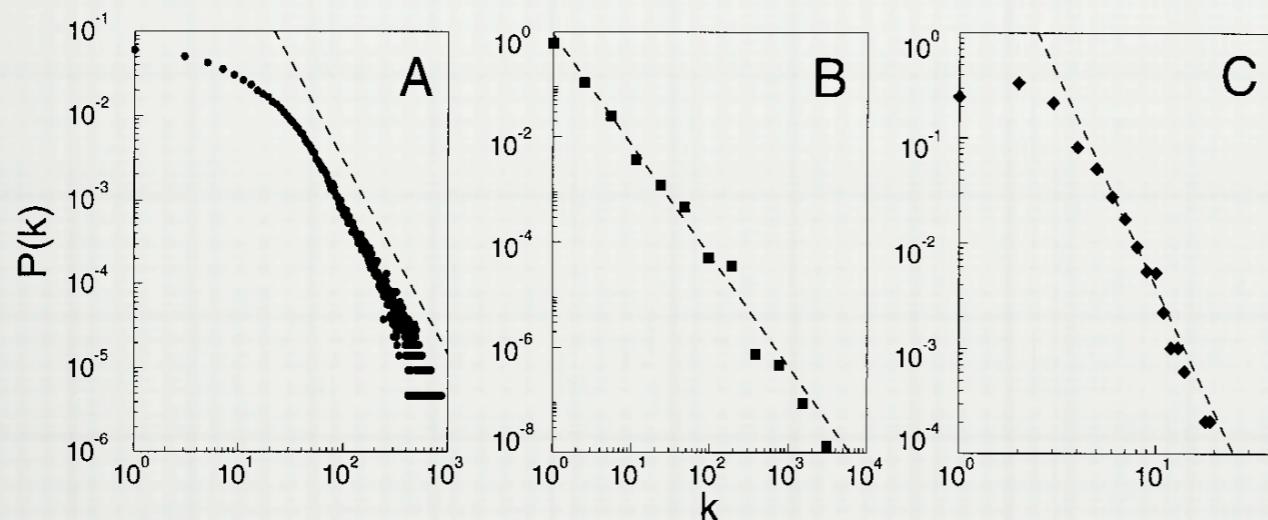


# СТРУКТУРА ВЕБ-ГРАФА



- "GRAPH STRUCTURE IN THE WEB" ANDREJ BRODER, RAVI KUMAR, ET AL. 2000.

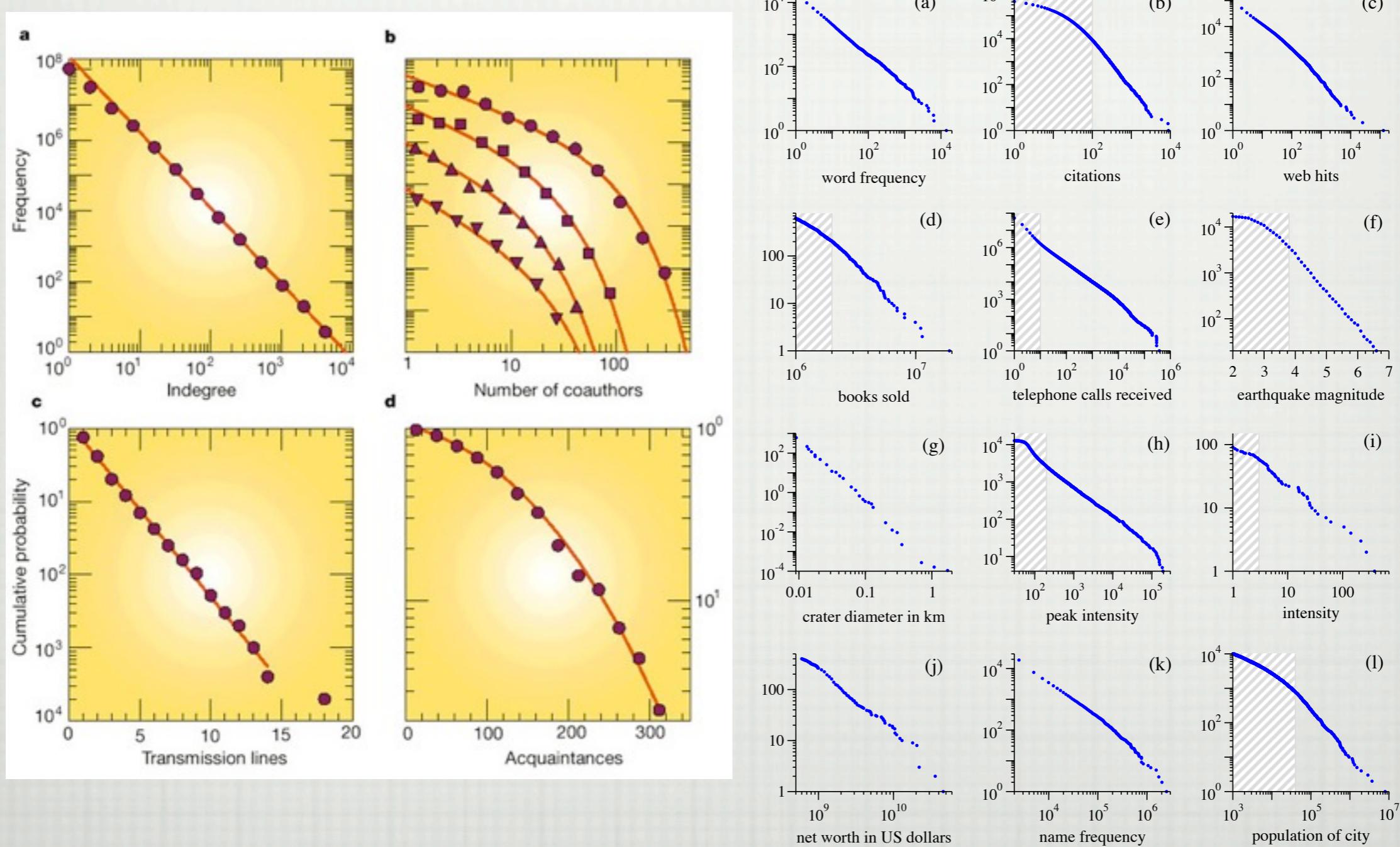
# СТЕПЕННОЙ ЗАКОН РАСПРЕДЕЛЕНИЯ



$$P(k) = Ck^{-\gamma}$$

**Fig. 1.** The distribution function of connectivities for various large networks. (A) Actor collaboration graph with  $N = 212,250$  vertices and average connectivity  $\langle k \rangle = 28.78$ . (B) WWW,  $N = 325,729$ ,  $\langle k \rangle = 5.46$  (6). (C) Power grid data,  $N = 4941$ ,  $\langle k \rangle = 2.67$ . The dashed lines have slopes (A)  $\gamma_{\text{actor}} = 2.3$ , (B)  $\gamma_{\text{www}} = 2.1$  and (C)  $\gamma_{\text{power}} = 4$ .

# МАСШТАБНО ИНВАРИАНТНЫЕ СЕТИ



# ЭФФЕКТ “МАЛОГО МИРА”



© Al Satterwhite

## An Experimental Study of the Small World Problem\*

JEFFREY TRAVERS

Harvard University

AND

STANLEY MILGRAM

The City University of New York

*Arbitrarily selected individuals ( $N=296$ ) in Nebraska and Boston are asked to generate acquaintance chains to a target person in Massachusetts, employing “the small world method” (Milgram, 1967). Sixty-four chains reach the target person. Within this group the mean number of intermediaries between starters and targets is 5.2. Boston starting chains reach the target person with fewer intermediaries than those starting in Nebraska; subpopulations in the Nebraska group do not differ among themselves. The funneling of chains through sociometric “stars” is noted, with 48 per cent of the chains passing through three persons before reaching the target. Applications of the method to studies of large scale social structure are discussed.*

- “THE SMALL-WORLD PROBLEM”. STANLEY MILGRAM. 1967.
- “AN EXPERIMENTAL STUDY OF THE SMALL WORLD PROBLEM”, J. TRAVERS, S. MILGRAM, 1969

# ЭКСПЕРИМЕНТ 1969

- 296 VOLUNTEERS, 217 SENT
- 196 NEBRASKA (1300 MILES)
- 100 BOSTON (25 MILES)
- TARGET IN BOSTON



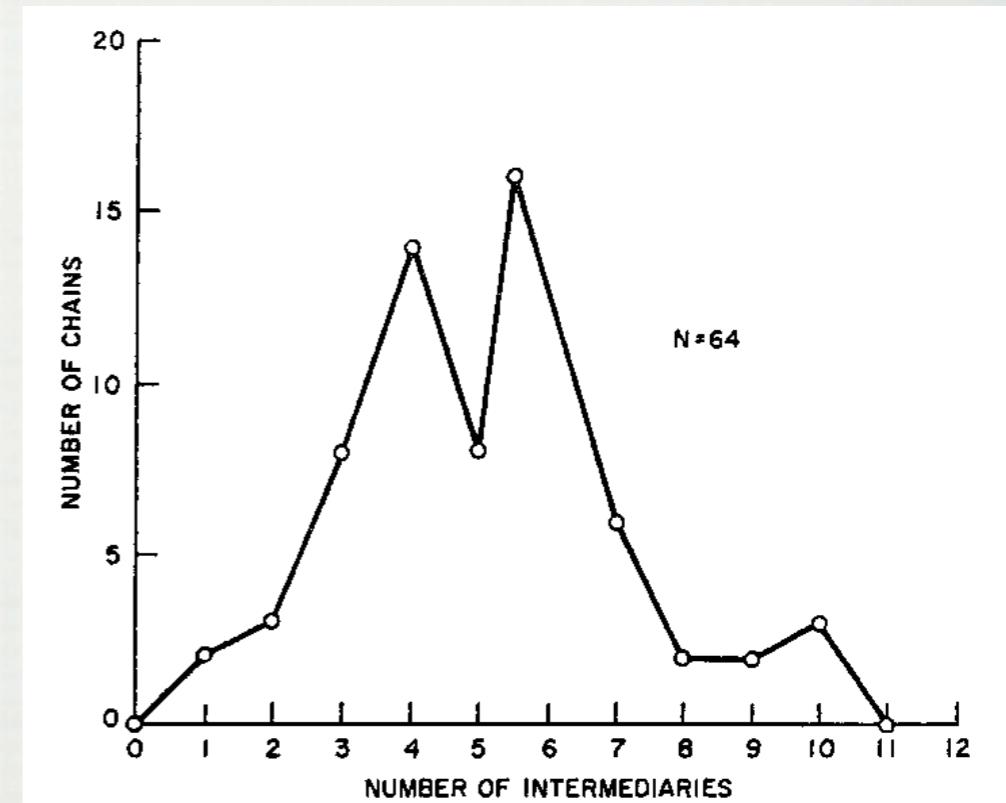
## HOW TO TAKE PART IN THIS STUDY

1. ADD YOUR NAME TO THE ROSTER AT THE BOTTOM OF THIS SHEET, so that the next person who receives this letter will know who it came from.
2. DETACH ONE POSTCARD. FILL IT OUT AND RETURN IT TO HARVARD UNIVERSITY. No stamp is needed. The postcard is very important. It allows us to keep track of the progress of the folder as it moves toward the target person.
3. IF YOU KNOW THE TARGET PERSON ON A PERSONAL BASIS, MAIL THIS FOLDER DIRECTLY TO HIM (HER). Do this only if you have previously met the target person and know each other on a first name basis.
4. IF YOU DO NOT KNOW THE TARGET PERSON ON A PERSONAL BASIS, DO NOT TRY TO CONTACT HIM DIRECTLY. INSTEAD, MAIL THIS FOLDER (POSTCARDS AND ALL) TO A PERSONAL ACQUAINTANCE WHO IS MORE LIKELY THAN YOU TO KNOW THE TARGET PERSON. You may send the folder to a friend, relative or acquaintance, but it must be someone you know on a first name basis.

- NAME, ADDRESS, OCCUPATION, JOB, HOMETOWN

# ЭКСПЕРИМЕНТ 1969

- REACHED THE TARGET  $N = 64$ , 29%
- AVE CHAIN LENGTH  $\langle L \rangle = 5.2$
- CHANNELS:
  - HOMETOWN  $\langle L \rangle = 6.1$
  - BUSINESS CONTACTS  $\langle L \rangle = 4.6$
- LOCATION:
  - BOSTON  $\langle L \rangle = 4.4$
  - NEBRASKA  $\langle L \rangle = 5.7$

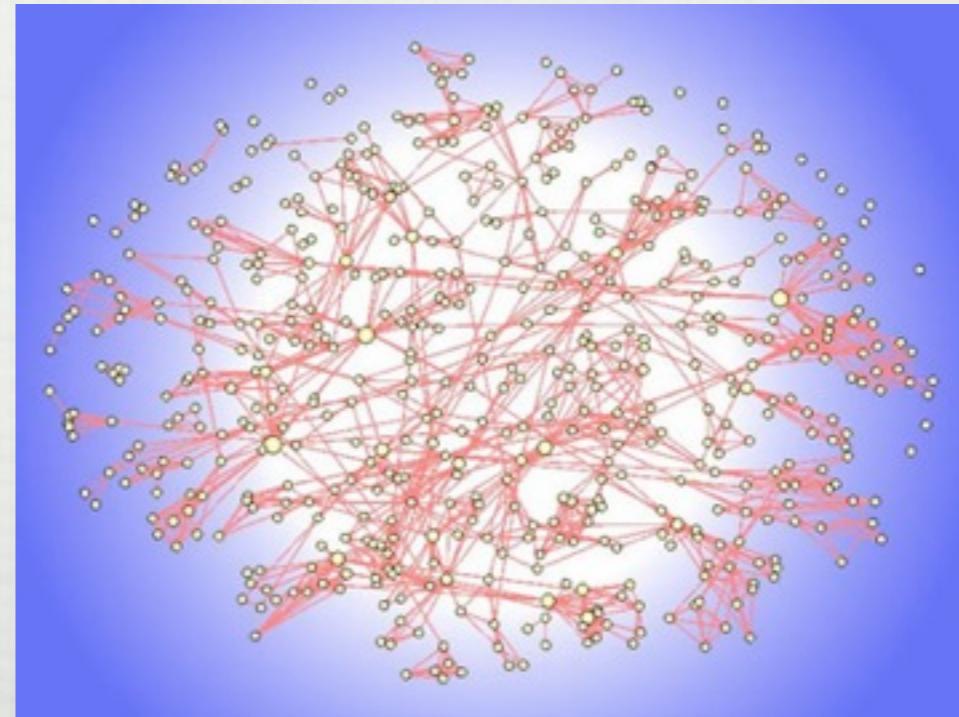


DEMONSTRATED THE FEASIBILITY  
OF "SMALL WORLD"  
INTERCONNECTEDNESS IN LARGE  
SOCIETY

# SIX DEGREES OF SEPARATION

- ТЕОРИЯ ШЕСТИ РУКОПОЖАТИЙ
- DUNCAN WATTS, 2001, EMAIL, 48,000 SENDERS,  $\langle L \rangle \sim 6$
- JURE LESKOVEC AND ERIC HORVITZ, 2007, MSN MESSENGER 240 MLN USERS,  $\langle L \rangle = 6.6$  USERS
- YAHOO, 2011, "YAHOO RESEARCH SMALL WORLD EXPERIMENT" ON FACEBOOK :)

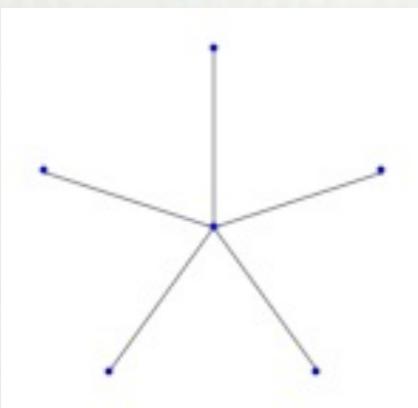
GRAPH DIAMETER  $D$   
AVE PATH LENGTH  $\langle L \rangle$



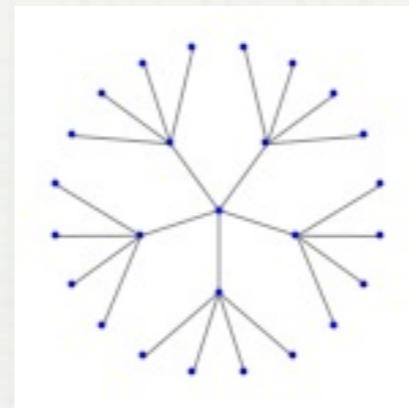
LOTHAR KREMPEL, CO-AUTHORSHIP NETWORK

# ПРОСТОЙ ПРИМЕР

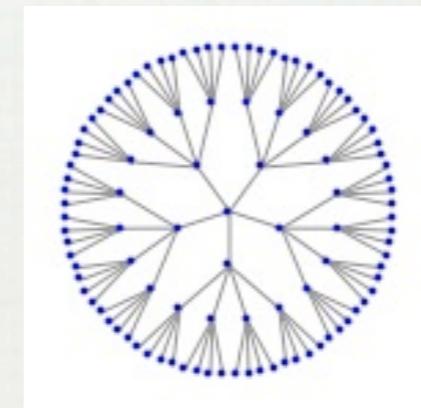
CAYLEY TREE (MOORE GRAPH)



6



26



106

У КАЖДОГО 6 ДРУЗЕЙ

$$D^k = N$$

$$k = \log N / \log D$$

6 МЛРД

50 ДРУЗЕЙ

$$k \sim 5.8$$

$$1 + d \sum_{i=0}^{k-1} (d-1)^i.$$

# Статистика

	Network	Type	$n$	$m$	$z$	$\ell$	$\alpha$	$C^{(1)}$	$C^{(2)}$	$r$	Ref(s).
Social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	[20, 415]
	company directors	undirected	7 673	55 392	14.44	4.60	—	0.59	0.88	0.276	[105, 322]
	math coauthorship	undirected	253 339	496 489	3.92	7.57	—	0.15	0.34	0.120	[107, 181]
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	—	0.45	0.56	0.363	[310, 312]
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	—	0.088	0.60	0.127	[310, 312]
	telephone call graph	undirected	47 000 000	80 000 000	3.16	—	2.1	—	—	—	[8, 9]
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0	—	0.16	—	[136]
	email address books	directed	16 881	57 029	3.38	5.22	—	0.17	0.13	0.092	[320]
	student relationships	undirected	573	477	1.66	16.01	—	0.005	0.001	-0.029	[45]
	sexual contacts	undirected	2 810	—	—	—	3.2	—	—	—	[264, 265]
Information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	-0.067	[14, 34]
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7	—	—	—	[74]
	citation network	directed	783 339	6 716 198	8.57	—	3.0/—	—	—	—	[350]
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	—	0.13	0.15	0.157	[243]
	word co-occurrence	undirected	460 902	17 000 000	70.13	—	2.7	—	0.44	—	[119, 157]
Technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	-0.189	[86, 148]
	power grid	undirected	4 941	6 594	2.67	18.99	—	0.10	0.080	-0.003	[415]
	train routes	undirected	587	19 603	66.79	2.16	—	—	0.69	-0.033	[365]
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	-0.016	[317]
	software classes	directed	1 377	2 213	1.61	1.51	—	0.033	0.012	-0.119	[394]
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	-0.154	[155]
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	-0.366	[6, 353]
Biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	-0.240	[213]
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	-0.156	[211]
	marine food web	directed	135	598	4.43	2.05	—	0.16	0.23	-0.263	[203]
	freshwater food web	directed	92	997	10.84	1.90	—	0.40	0.48	-0.326	[271]
	neural network	directed	307	2 359	7.68	3.97	—	0.18	0.28	-0.226	[415, 420]