Social Power: Centrality Measures

Leonid E. Zhukov

National Research University Higher School of Economics

13.02.2012

Centrality Measures

Sociology. Freeman, 1979.
Most "important"actors: actor location in the social network

- Actor Centrality - involvment with other actors, many ties, source or recepient
- Actor Prestige - recipient (object) of many ties, ties directed to an actor

Three graphs:

- Star graph
- Circle graph
- Line graph

Centrality Measures

Degree Centrality

$$C_D(i) = k(i) = \sum_j A_{ij} = \sum_j A_{ji}$$

Normalized degree centrality

$$C_D^*(i) = \frac{1}{n-1} C_D(i)$$

High centrality degree -direct contact with many other actors
Low degree - not active, peripherial

Centrality Measures

Closeness Centrality
How close an actor to all the other actors in network

$$C_C(i) = \left[\sum_j d(i,j)\right]^{-1}$$

Normalized closenss centrality

$$C_C^*(i) = (n-1)C_C(i)$$

Actor in the center can quickly interact with all others, short communication path to others, minimal number of steps to reach others

Centrality Measures

Betweenness Centrality
Number of shortest paths going through the actor $\sigma_{st}(i)$

$$C_B(i) = \sum_{s \neq t \neq i} \frac{\sigma_{st}(i)}{\sigma_{st}}$$

Normalized closenss centrality

$$C_B^*(i) = \frac{2}{(n-1)(n-2)} C_B(i)$$

Probability that a communication from s to t will go through i (geodesics)
Edge betweenness

Centrality Measures

Degree Prestige

$$P_D(i) = k_{in}(i) = \sum_j A_{ji}$$

Normalized degree prestige

$$P_D^*(i) = \frac{1}{n-1} P_D(i)$$

Prestigious actors recieve many nominations

Centrality Measures

Proximity Prestige

Influence domain - set of actors that can reach i directly and indirectly.

I_i - size of influence domain. Average distance $\sum_j d(j,i)/I_i$

$$P_p(i) = \frac{I_i/(n-1)}{\sum_j d(j,i)/I_i}$$

Centrality Measures

Status/Rank Prestige. Eigenvector Centrality
Not only the number of direct connections or size of influence domain, but prestige of those actors

$$p_i \leftarrow \sum_{j \in N(i)} p_j = \sum_j A_{ji} p_j$$

$$p_i = \frac{1}{\kappa} \sum_j A_{ji} p_j$$

$$\mathbf{A}^T \mathbf{p} = \kappa \mathbf{p}$$

Eigenproblem, solution $\kappa = \lambda_{max}$, $\mathbf{p} = \mathbf{p_{max}}$

Centrality Measures

Centralization - how central the most central node in relation to all other nodes.

$$C_x = \frac{\sum_i^N [C_x(p_*) - C_x(p_i)]}{\max \sum_i^N [C_x(p_*) - C_x(p_i)]}$$

C_x - one of the centrality measures