Link Analysis

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Link Analysis

Eigenvalue Prestige
An actor gives equal parts of its prestige to all nearest neighbours

$$p_i \leftarrow \sum_j A_{ji} \frac{p_j}{k_{out}(j)}$$

$$\mathbf{p} = (\mathbf{D}^{-1}\mathbf{A})^T \mathbf{p}$$

where $\mathbf{D}_{ii} = max(k_i, 1)$
$(\mathbf{D}^{-1}\mathbf{A})^T$ - stochastic matrix, guaranteed $\lambda_{max} = 1$

$$(\mathbf{D}^{-1}\mathbf{A})^T \mathbf{p} = \lambda \mathbf{p}$$

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PageRank (Webgraph). Brin and Page, 1998

- zero out degree nodes, $k_{out}(i) = 0$
- zero in degree nodes, $k_{in}(i) = 0$

Transition matrix:

$$\mathbf{P} = \mathbf{D}^{-1}\mathbf{A}$$

Stochastic matrix:

$$\mathbf{P}' = \mathbf{P} + \frac{\mathbf{d}^T \mathbf{e}}{n}$$

PageRank matrix:

$$\mathbf{P}'' = \alpha \mathbf{P}' + (1 - \alpha)\frac{\mathbf{e}^\mathbf{T} \mathbf{e}}{n}$$

Link Analysis

Eigenvalue problem:

$$\mathbf{P}''^{T}\mathbf{p} = \lambda \mathbf{p}$$

Perron-Frobenius theorem:

- stochastic (non-negative and rows sum up to one)
- irreducible (strongly connected graph)
- aperiodic

then unique largest eigenvalue $\lambda_{max} = 1$, positive eigenvector and power iterations coneverge to it. Solution satisfies $|\mathbf{p}|_1 = 1$

$$\lambda \mathbf{p} = \alpha \left((\mathbf{D}^{-1}\mathbf{A})^T + \frac{\mathbf{e}^T\mathbf{d}}{n} \right) \mathbf{p} + (1-\alpha)\frac{\mathbf{e}}{n}$$

Link Analysis

Hubs and Authorities (HITS) . Kleinberg, 1999
Citation networks. Reviews vs original research (authoritative) papers

- authorities, contain useful information, a_i
- hubs, contains links to authorities, h_i

Mutual recursion

- Good authorities reffered by good hubs

$$a_i \leftarrow \sum_j A_{ji} h_j$$

- Good hubs point to good authorities

$$h_i \leftarrow \sum_j A_{ij} a_j$$

Link Analysis

Hubs and Authorities (HITS)

$$\begin{aligned} \mathbf{a} &= \alpha \mathbf{A}^T \mathbf{h} \\ \mathbf{h} &= \beta \mathbf{A} \mathbf{a} \end{aligned}$$

Symmetric eigenvalue problem

$$\begin{aligned} (\mathbf{A}^T \mathbf{A})\mathbf{a} &= \lambda \mathbf{a} \\ (\mathbf{A} \mathbf{A}^T)\mathbf{h} &= \lambda \mathbf{h} \end{aligned}$$

where eigenvalue $\lambda = (\alpha\beta)^{-1}$