Complex Networks

Leonid E. Zhukov

School of Applied Mathematics and Information Science
National Research University Higher School of Economics

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Class Technicalities

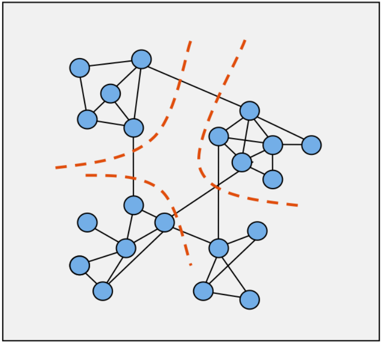
- 10 lectures, 5 homeworks, final exam
- Website: http://www.leonidzhukov.ru/hse/2013/socialnetworks
- My email: lzhukov@hse.ru
- Programming: Matlab, Octave, R, Python,
- Matlab libraries: MatlabBGL, GraphViz (graphviz4matlab, graphviz)
- Python libraries: NetworkX, iGraph
- Visusualization: yEd, Gephi

Lectures

1. Intorduction to complex networks
2. Power law distributions
3. Random graphs
4. Dynamical growth models
5. Graph centrality metrics
6. Link analysis
7. Structural equivalence
8. Network communities
9. Network community detectinion algorithms
10. Graph partitioning algorithms

Terminolgy

- network = graph
- nodes = vertices, actors
- links = edges, relations
- clusters = communities



Algorithms & Data Structures

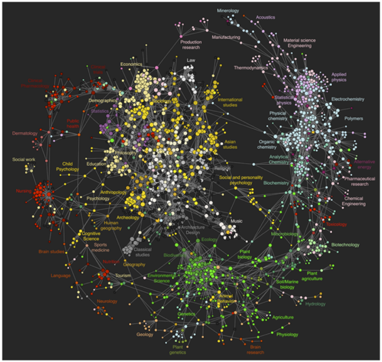
- Breadth first seatch (BFS), Depth First Search (DFS)
- Connected components
- Shortest paths

- Linked lists
- Sparse matrix

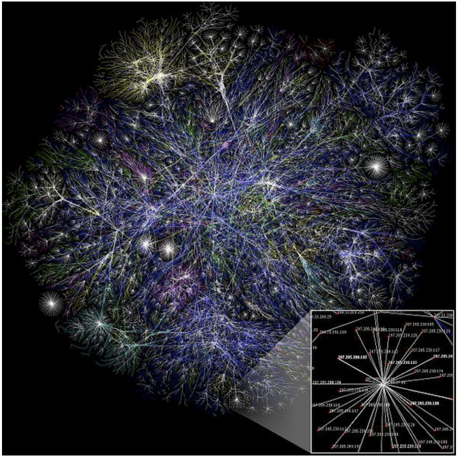
- Matrix, vector operations
- Eigenvalues and Eigenvectors ($\mathbf{Ax} = \lambda \mathbf{x}$)

Complex networks

- Networks = nodes + edges
- not regular, but not random
- complex (nontrivial) topolgy
- universal properties
- everywhere
- independent agents?

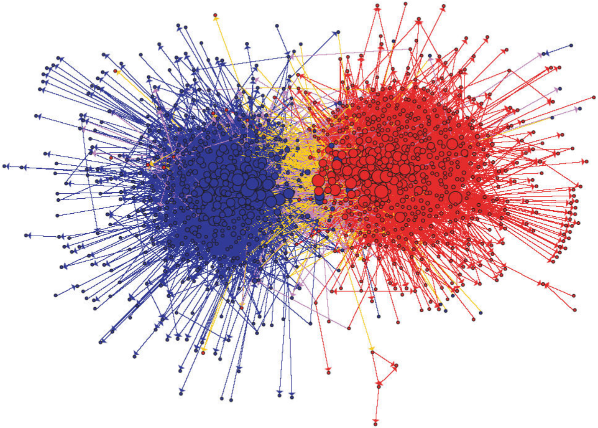


Examples: Internet

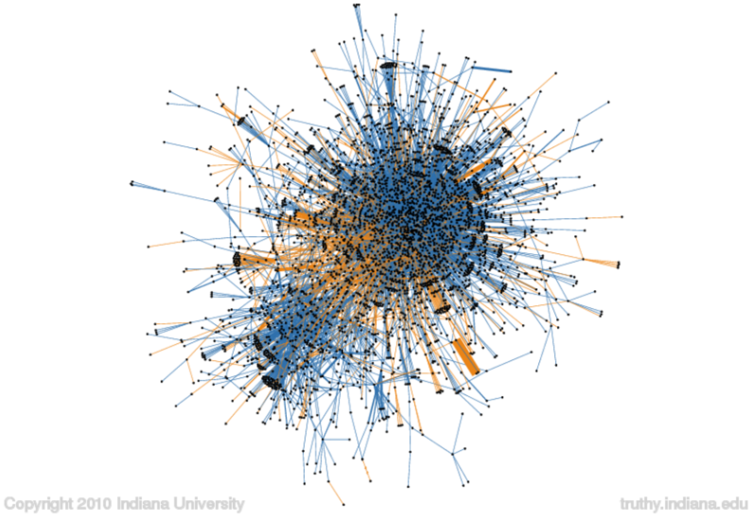


Barrett Lyon, OPTE.org, class C networks

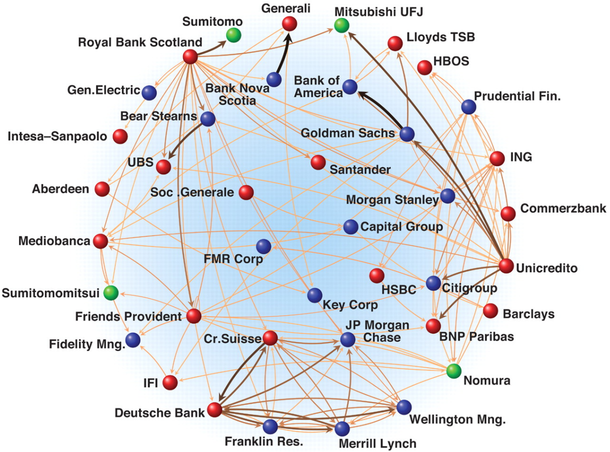
Examples: Political blogs



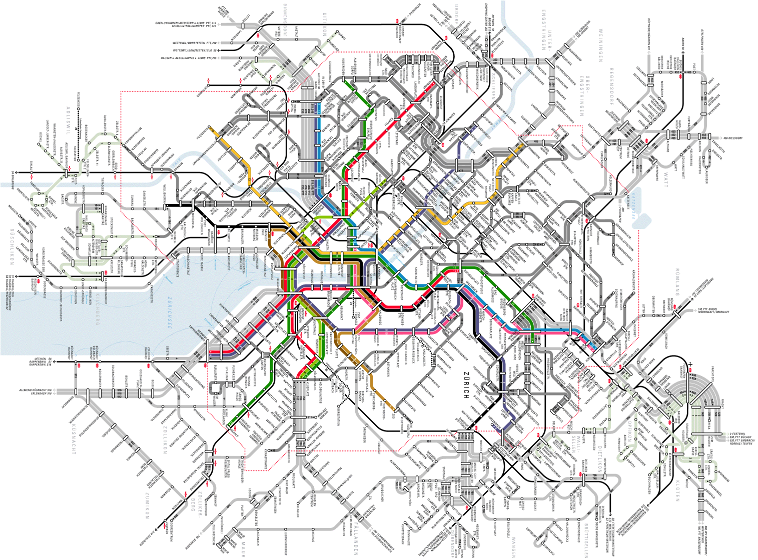
Examples: Twitter



Examples: Finance

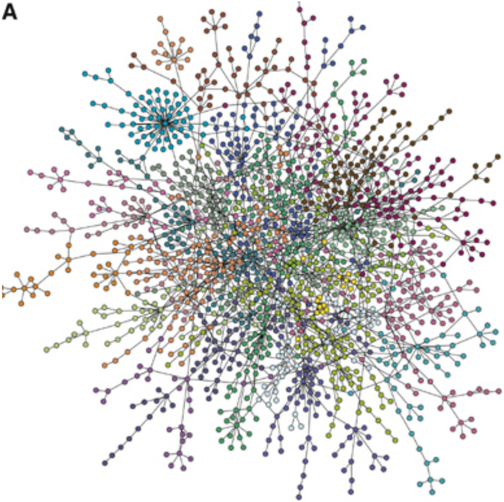


Examples: Transportation

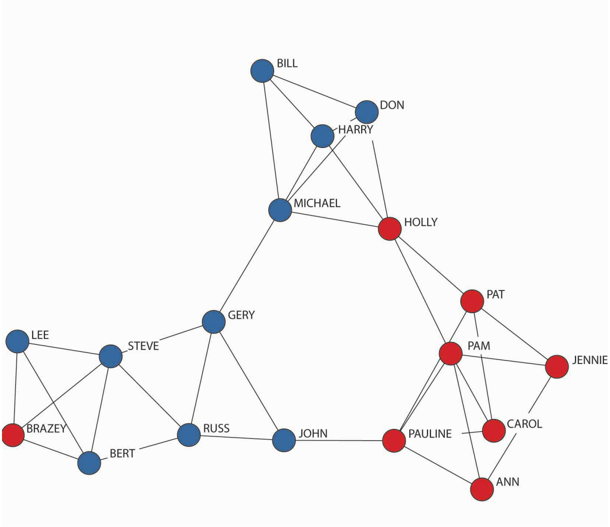


Examples: Biology

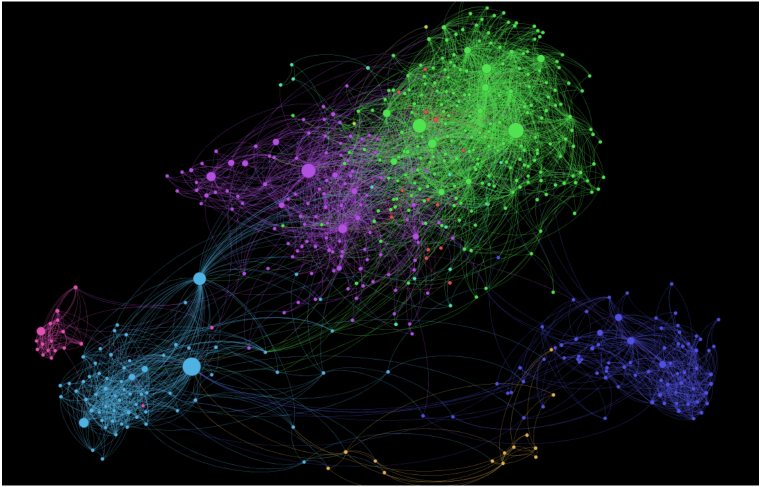
Protein-protein interactions



Examples: Organization



Examples: Facebook communities tructure



Examples: Facebook

Social graph 500mln people, Paul Butler, 2010

Network structure

- if nodes connected, how far the nodes on the network
- are some nodes more important than others
- communities (clusters)
- macroscopic (statistical) properties
- microscopic (local) structure
- resilience to attacks
- formation and growth

Processes on networks

- Epidemics
- Diffusion of information
- Spread of influence
- Collective behaviour
- Network economy

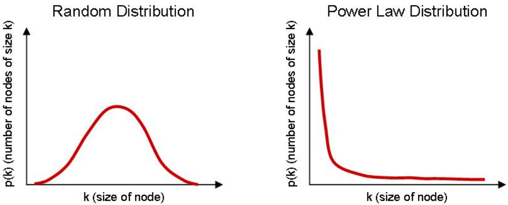
Network models

- Random graph (Erdos & Renyi, 1956)
- Small world models (Watts & Strogatz, 1998)
- Preferential attachement (Barabasi & Albert, 1999)
- Strategic models (Jackson & Wolinksy, 1996)

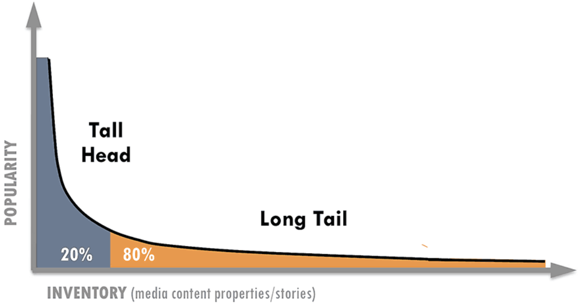
Macroscopic properties

- Heavy tail (long tail) degree distributions
- Small world
- High clustering
- Giant connected component

Long tail

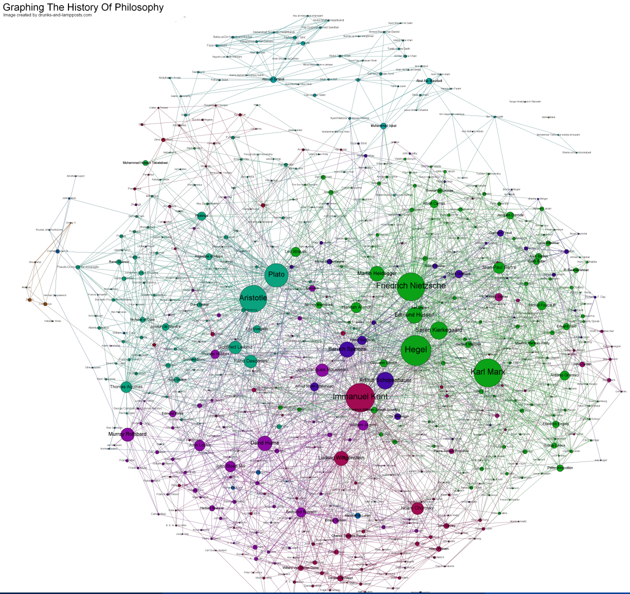


Random Distribution
p(k) (number of nodes of size k)
k (size of node)
Power Law Distribution
p(k) (number of nodes of size k)
k (size of node)

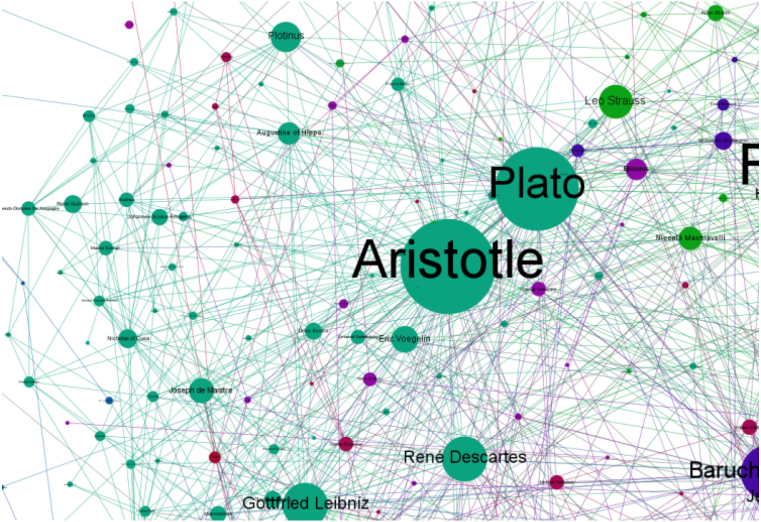


POPULARITY
Tall
Head
Long Tail
20%
80%
INVENTORY (media content properties/stories)

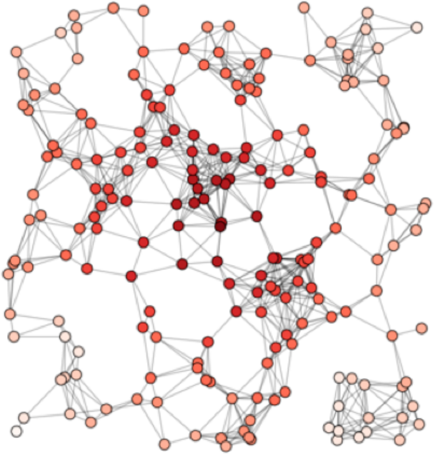
Power law


Graphing The History Of Philosophy
Image created by drunks-and-lampposts.com
Plato
Aristotle
Friedrich Nietzsche
Hegel
Karl Marx
Immanuel Kant
Martin Heidegger
Edmund Husserl
Søren Kierkegaard
Baruch Spinoza
Arthur Schopenhauer
Jean-Jacques Rousseau
Gottfried Leibniz
René Descartes
Thomas Aquinas
Murray Rothbard
David Hume
John Locke
John Stuart Mill
Ludwig Wittgenstein
Bertrand Russell
Noam Chomsky
Jean-Paul Sartre

Power law

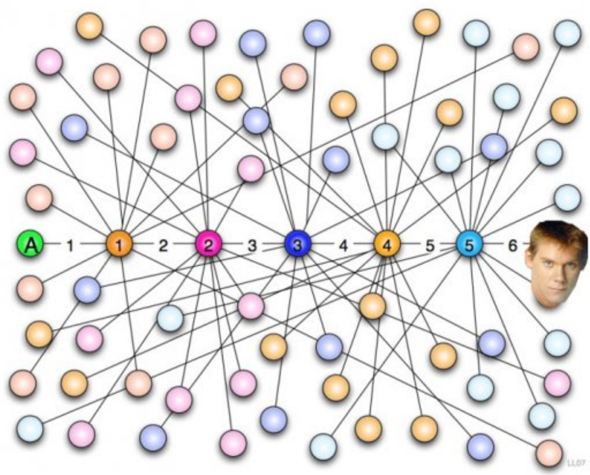

Plotinus
Leo Strauss
Augustine of Hippo
Plato
Epicurus
Aristotle
Niccolò Machiavelli
Eric Voegelin
Nicholas of Cusa
Joseph de Maistre
René Descartes
Baruch
Gottfried Leibniz

High clustering



Giant connected component

Six degrees of separation



Six degrees of separation

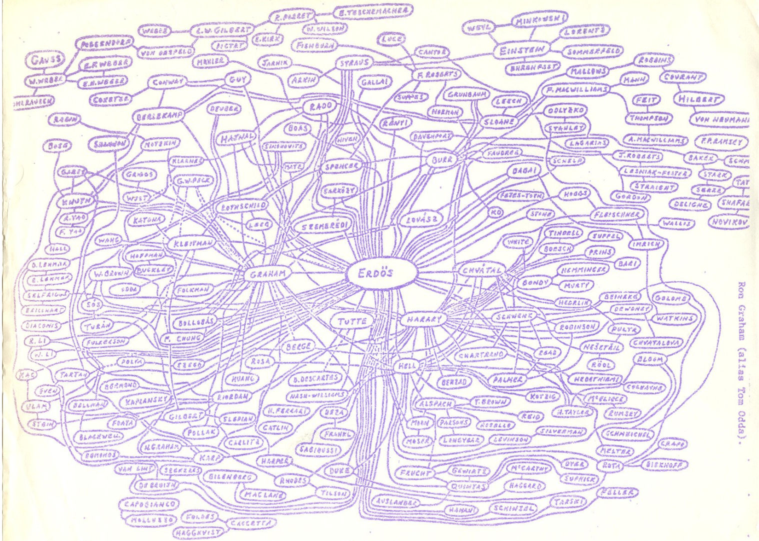


Figure 1

To appear in Topics in Graph Theory (F. Harary, ed.) New York Academy of Sciences (1979).

Small world

© Al Satterwhite

An Experimental Study of the Small World Problem*

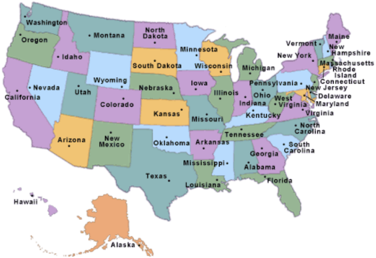
JEFFREY TRAVERS
Harvard University
AND
STANLEY MILGRAM
The City University of New York

Arbitrarily selected individuals (N=296) in Nebraska and Boston are asked to generate acquaintance chains to a target person in Massachusetts, employing "the small world method" (Milgram, 1967). Sixty-four chains reach the target person. Within this group the mean number of intermediaries between starters and targets is 5.2. Boston starting chains reach the target person with fewer intermediaries than those starting in Nebraska; subpopulations in the Nebraska group do not differ among themselves. The funneling of chains through sociometric "stars" is noted, with 48 per cent of the chains passing through three persons before reaching the target. Applications of the method to studies of large scale social structure are discussed.

- "The small-world problem". Stanley Milgram, 1967
- "An experimental study of the small world problem J. Travers, S. Milgram, 1969

Stanely Milgram's 1969 experiment

- 296 volunteers, 217 sent
- 196 in Nebraska (1300 miles)
- 100 in Boston (25 miles)
- Target in Boston
- Name, address, occupation, job, hometowm



Stanely Milgram's 1969 experiment

HOW TO TAKE PART IN THIS STUDY

1. ADD YOUR NAME TO THE ROSTER AT THE BOTTOM OF THIS SHEET, so that the next person who receives this letter will know who it came from.

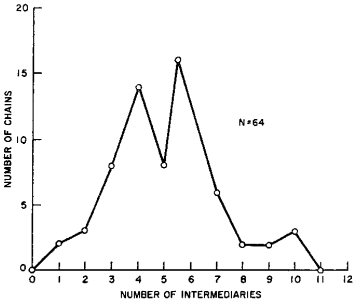
2. DETACH ONE POSTCARD. FILL IT OUT AND RETURN IT TO HARVARD UNIVERSITY. No stamp is needed. The postcard is very important. It allows us to keep track of the progress of the folder as it moves toward the target person.

3. IF YOU KNOW THE TARGET PERSON ON A PERSONAL BASIS, MAIL THIS FOLDER DIRECTLY TO HIM (HER). Do this only if you have previously met the target person and know each other on a first name basis.

4. IF YOU DO NOT KNOW THE TARGET PERSON ON A PERSONAL BASIS, DO NOT TRY TO CONTACT HIM DIRECTLY. INSTEAD, MAIL THIS FOLDER (POSTCARDS AND ALL) TO A PERSONAL ACQUAINTANCE WHO IS MORE LIKELY THAN YOU TO KNOW THE TARGET PERSON. You may send the folder

Stanely Milgram's 1969 experiment

- Reached the target N = 64, 29%
- Average chain length <L> = 5.2
- Channeles: hometown <L> = 6.1
- Business contacts <L> = 4.6
- From Boston <L> = 4.4
- From Nebraska <l> = 5.7



6 degrees

- Duncal Watts, 2001, Email, 48,000 senders, <L> 6
- Jure Lescovec et al, 2007, MSN Messenger graph, 240mln users, <L> = 6.6
- Facebook
- Moore Graph (Cayley tree) d friends $d^K = n$ $k = logn/logd$
  6 bln, 50 friends, k 5.8

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