Dynamical growth models

Leonid E. Zhukov

School of Applied Mathematics and Information Science
National Research University Higher School of Economics

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НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ

Preferential attachment

Barabasi and Albert, 1999
Dynamical (growth) model

- $t = 0$, n_0 unconnected nodes
- growth: on every step add a node with m edges ($m \leq n_0$)
- Preferential attachment: probability of linking to existing node is proportional to the node degreek_i

$$\Pi(k_i) = \frac{k_i}{\sum_i k_i}$$

after t steps: $N = n_0 + t$ nodes, $M = mt$ edges
nodes appear one at a time, node "i"at t_i, then $k_i(t_i) =$?

Preferential attachment

mean field approximation, distribution of average (expected) node degrees, continues approximation

$$k_i(t+\delta t) = k_i(t) + m\Pi(k_i)\delta t$$

$$\frac{dk_i(t)}{dt} = m\Pi(k_i) = m\frac{k_i}{\sum_i k_i} = \frac{mk_i}{2mt}$$

$$\frac{dk_i(t)}{dt} = \frac{k_i(t)}{2t}$$

initial conditions: $k_i(t_i) = m$

Preferential attachment

Time evolution of a node degree

$$k_i(t) = m \left(\frac{t}{t_i} \right)^{1/2}$$

Distribution function:

$$P(k) = P(k_i(t) = k) = \frac{d}{dk} P(k_i(t) < k)$$

CDF

$$P(k_i(t) < k) = P(t_i \geq \frac{m^2}{k^2} t) =$$

$$= \frac{N_{t_i > ..}}{N_t} = \frac{n_0 + t - m^2 t / k^2}{n_0 + t} \approx 1 - \frac{m^2}{k^2}$$

Preferential attachment

- "power law"distribution function

$$P(k) = \frac{2m^2}{k^3}$$

- Average path length

$$\langle l \rangle \sim \log(N)/\log(\log(N))$$

- Clustering coefficient

$$C \sim N^{-0.75}$$

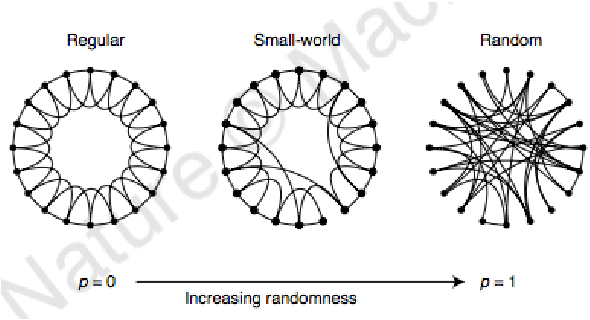
Small world

Watts and Strogatz, 1998

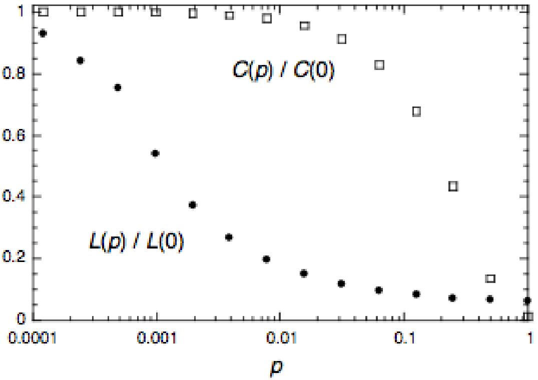
One parameter model, interpolation between regular lattice and random graph

- start with regular lattice with N nodes, K nearest neighbours (node degree)
- randomly connect with other nodes with probability p, forms $pNK/2$ "long distance"connections
- $p = 0$ regular lattice, $p = 1$ random graph

Small world

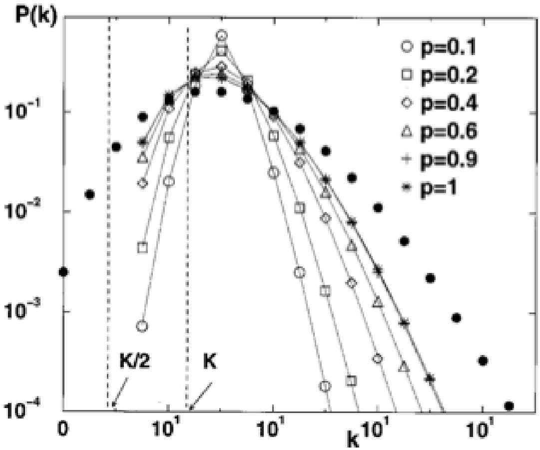


Small world



p- parameter, $\langle l(p) \rangle$, $C(p)$
$p \to 0$, ring lattice, $\langle l(0) \rangle = 2N/K$, $C(0) = 3/4$
$p \to 1$, random graph, $\langle l(1) \rangle \sim \log(N)/\log(K)$, $C(1) \sim K/N$

Small world



Model comparison

	Random	BA model	WS model	Real World
P(k)	$\frac{z^k e^{-z}}{k!}$	k^{-3}	poisson like	power law
C	$p = \frac{\langle k \rangle}{N}$	$N^{-0.75}$	const	large
$\langle l \rangle$	$\frac{log(N)}{log(\langle k \rangle)}$	$\frac{log(N)}{loglog(N)}$	$log(N)$	small world

References

- Collective dynamics of ‘small-world’ networks. Duncan J. Watts and Steven H. Strogatz. Nature 393 (6684): 440–442, 1998
- Emergence of Scaling in Random Networks, AL Barabasi and R. Albert, Science 286, 509–512, 1999