Information cascades

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НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ

Observational learning

- Learnig by observing the past decisions of others (social learning)
- Convergence to similar behavior
- Could have similar information, similar alternatives, similar payoff
- Make dicision by watching others
- Observable actions vs observable signals
- Rational decision making
- Spread of fashion, fads, music hits, techonology adoptions
- There is no "true" learning, behavior is imitative

"A simple model of herd behavior", Abhijit Banerjee, 1992
"A Theory of Fads, Fashion, Custom, and Cultural Change as Information Cascades", S. Bikhchandani, D Hirshleifer and I.Welch, 1992

Information cascade

- Group of people, sequential decision making
- Every person makes decision once from limited action space with incomplete information
- Every person observes decison of others before him before making own decision
- Decision is made based on private information (signal) + observations of decision of others
- Information cascade: do what everone else does even against private information

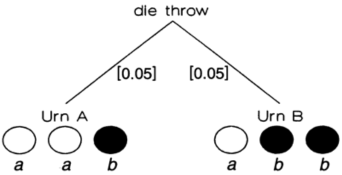
Baysian learning

- Hypothesis testing: H_1, H_2
- Apriory probability: $P(H_1)$, $P(H_2)$
- Bayes's rule:

$$
\begin{aligned}
P(H_1|E) &= \frac{P(E|H_1)P(H_1)}{P(E)} \\
P(H_2|E) &= \frac{P(E|H_2)P(H_2)}{P(E)} \\
P(E) &= P(E|H_1)P(H_1) + P(E|H_2)P(H_2)
\end{aligned}
$$

- Aposteriory: $P(H_1|E)$, $P(H_2|E)$

Simlpe experiment



- Urns A, B:
  $P(A) = P(B) = 1/2$
- Marbles a,b:
  $P(a|A) = 2/3,\ P(b|A) = 1/3$
  $P(b|A) = 1/3,\ P(b|B) = 2/3$

Simlpe experiment

- 1. Selected "b-marble"

$$
\begin{aligned}
P(B|b) &= \frac{P(b|B)P(B)}{P(b)} \\
P(b) &= P(b|B)P(B) + P(b|A)P(A) = \\
&= 2/3 \cdot 1/2 + 1/3 \cdot 1/2 = 1/2 \\
P(B|b) &= \frac{2/3 \cdot 1/2}{1/2} = 2/3
\end{aligned}
$$

Rational choice "B -urn"

Simlpe experiment

- 2a. Selected "b-marble"

$$
\begin{aligned}
P(B|b,b) &= \frac{P(b,b|B)P(B)}{P(b,b)} \\
P(b,b|B) &= P(b|B)P(b|B) = 2/3 \cdot 2/3 = 4/9 \\
P(b,b|A) &= P(b|A)P(b|A) = 1/3 \cdot 1/3 = 1/9 \\
P(b,b) &= P(b,b|B)P(B) + P(b,b|A)P(A) = \\
&= 4/9 \cdot 1/2 + 1/9 \cdot 1/2 = 5/18 \\
P(B|b,b) &= \frac{4/9 \cdot 1/2}{5/18} = 4/5
\end{aligned}
$$

Rational choice "B-urn"

Simlpe experiment

- 2b. Selected "a-marble"

$$
\begin{aligned}
P(B|b,a) &= \frac{P(b,a|B)P(B)}{P(b,a)} \\
P(b,a|B) &= P(b|B)P(a|B) = 2/3 \cdot 1/3 = 2/9 \\
P(b,a|A) &= P(b|A)P(b|A) = 1/3 \cdot 2/3 = 2/9 \\
P(b,a) &= P(b,a|B)P(B) + P(b,a|A)P(A) = \\
&= 2/9 \cdot 1/2 + 2/9 \cdot 1/2 = 2/9 \\
P(B|b,a) &= \frac{2/9 \cdot 1/2}{2/9} = 1/2
\end{aligned}
$$

Rational choice to follow own signal, "A-urn"

Simlpe experiment

- 3. Selected "a-marble" (1 step- "B-urn", 2 step -"B-urn")

$$
\begin{aligned}
P(B|b,b,a) &= \frac{P(b,b,a|B)P(B)}{P(b,b,a)} \\
P(b,b,a|B) &= P(b|B)P(b|B)P(a|B) = 2/3 \cdot 2/3 \cdot 1/3 = 4/27 \\
P(b,b,a|A) &= P(b|A)P(b|A)P(a|A) = 1/3 \cdot 1/3 \cdot 2/3 = 2/27 \\
P(b,b,a) &= P(b,b,a|B)P(B) + P(b,b,a|A)P(A) = \\
&= 4/27 \cdot 1/2 + 2/27 \cdot 1/2 = 3/27 \\
P(B|b,b,a) &= \frac{4/27 \cdot 1/2}{3/27} = 2/3
\end{aligned}
$$

Rational choice "B-urn", inspite of own signal!

Simple experiment

TABLE 2—DATA FOR SELECTED PERIODS OF SESSION 2

Period	Urn used	Subject number: Urn decision (private draw)						Cascade outcome
		1st round	2nd round	3rd round	4th round	5th round	6th round	
5	B	S12: A (*a*)	S11: B (*b*)	S9: B (*b*)	S7: B (*b*)	**S8: B (*a*)**	**S10: B (*a*)**	cascade
6	A	S12: A (*a*)	S8: A (*a*)	**S9: A (*b*)**	**S11: A (*b*)**	S10: A (*a*)	S7: A (*a*)	cascade
7	B	S8: B (*b*)	S7: A (*a*)	S10: B (*b*)	S11: B (*b*)	S12: B (*b*)	**S9: B (*a*)**	cascade
8	A	S8: A (*a*)	S9: A (*a*)	S12: B* (*b*)	S10: A (*a*)	**S11: A (*b*)**	S7: A (*a*)	cascade
9	B	S11: A (*a*)	S12: A (*a*)	**S8: A (*b*)**	**S9: A (*b*)**	**S7: A (*b*)**	**S10: A (*b*)**	reverse cascade

Notes: Boldface—Bayesian decision, inconsistent with private information.
*—Decision based on private information, inconsistent with Bayesian updating.

General Cascade Model

- Group of people $\{1, .., n\}$ sequentially making decisions acceptiing/rejecting an option
- State of the world (random):
  'G' - good, 'B' - bad,
  $Pr[G] = p$, $Pr[B] = 1 - p$
- Payoff: $v_G > 0$, $v_B < 0$
  Expected payoff without any information $v_G p + v_B(1 - p) = 0$
- Private signal:
  'H' - accpeting is a good idea, 'L' - acceepting is a bad idea.
  Random, but truthful, $q > 1/2$
  $Pr[H|G] = q$, $Pr[L|G] = 1 - q$
  $Pr[H|B] = 1 - q$, $Pr[L|B] = q$

General Cascade Model

- No signal:

$$E^{no-signal}[payoff] = v_G Pr[G] + v_B Pr[B] = v_G p + v_B(1-p) = 0$$

- INdividual decisions: High signal 'H':

$$Pr[G|H] = \frac{Pr[H|G]Pr[G]}{Pr[H]} = \frac{qp}{qp + (1-q)(1-p)} > p$$
$$Pr[B|H] = \frac{Pr[H|B]Pr[B]}{Pr[H]} = \frac{(1-q)(1-p)}{qp + (1-q)(1-p)} < 1-p$$

$$E^{signal}[payoff] = v_G Pr[G|H] + v_B Pr[B|H] > E^{no-signal}[payoff]$$

- Rational agent should accept the option

General Cascade Model

- Mutliple signals $S = \{HLH..LHLL\}$, $a = \#H, b = \#L$
- Posterior probability:

$$Pr[G|S] = \frac{pq^a(1-q)^b}{pq^2(1-q)^b + (1-p)(1-q)^a q^b}$$

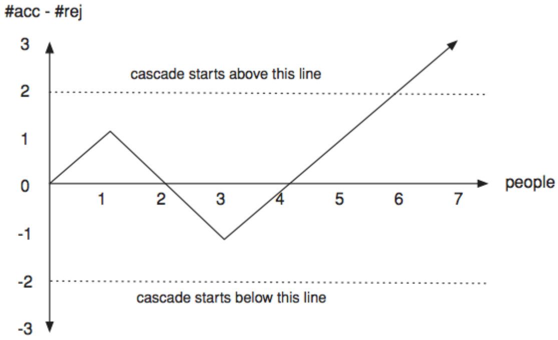
- if $a > b$, $Pr[G|S] > Pr[G]$
  if $a < b$, $Pr[G|S] < Pr[G]$
  if $a = b$, $Pr[G|S] = Pr[G] = p$
- Rational individual should accpect the option when gets more H signals than L

Sequential decision making

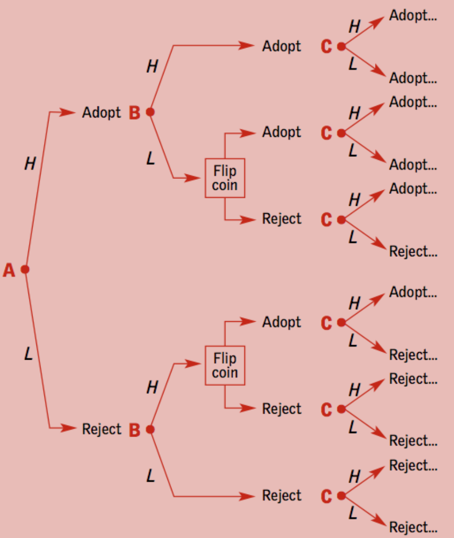
Each person can see the choice of previous people, but not their signals

1. Person 1. Follow private signal (1). Action reveals his private signal
2. Person 2. Follows 2 signals = his private (2) + private signal (1)
   if privite (2) = private (1), follows his private signal (1)
   if privite (2) $\neq$ private(1), follows his private
   Action reveals his private signal (2)
3. Person 3. Follows 3 signals = his private (3) + private (2) + private(1)
   if private (1) $\neq$ private (2), follows his private signal (3)
   if private (1) = private (2), follows signals (1), (2), not his private signal (3)
   Action *does not* reveal his private signal
4. Person 4 etc. If private (1) = private (2), follows signals (1), (2), not his private signal (4)

Information cascade



Information cascades



Source: Hirshleifer (1995)

Information cascades

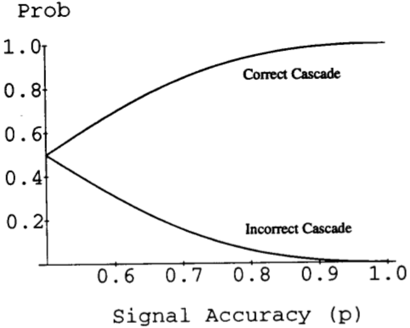
Let the true state of the world be 'G'. Probability of cascade after 2 people

- Probability of Up cascade:
  $Pr[HH] = q^2$,
  $Pr[HL] = q(1-q)$
  $Pr[Up\ cascade] = q^2 + q(1-q)1/2 = q(q+1)/2$
  $q = 0.5 \Rightarrow Pr = 37.5\%$, $q = 0.6 \Rightarrow Pr = 48\%$,
- Probability of No cascade:
  $Pr[HL] = q(1-q)$,
  $Pr[LH] = q(1-q)$
  $Pr[No\ cascade] = q(1-q)1/2 + q(1-q)1/2 = q(1-q)$
  $q = 0.5 \Rightarrow Pr = 25\%$, $q = 0.6 \Rightarrow Pr = 24\%$,
- Probability of Down cascade:
  $Pr[LL] = (1-q)^2$,
  $Pr[LH] = q(1-q)$
  $Pr[Down\ cascade] = (1-q)^2 + q(1-q)1/2 = (1-q)(2-q)/2$
  $q = 0.5 \Rightarrow Pr = 37.5\%$, $q = 0.6 \Rightarrow Pr = 28\%$,

Information cascades

Probability of cascade after n (even) people

- $Pr[No\ cascade] = (q - q^2)^{n/2}$
- $Pr[Up\ cascade] = \frac{q(q+1)(1-(q-q^2)^{n/2})}{2(1-q-q^2)}$
- $Pr[Down\ cascade] = \frac{(q-2)(q-1)(1-(q-q^2)^{n/2})}{2(1-q-q^2)}$



Information cascades

- Cascades very easy to start
- With large number of people a cascade happens almost surely $\lim_{n\to\infty}(q - q^2)^{n/2} \to 0$
- Cascades prevent information aggregation (start based on little information)
- Cascades can be wrong
- Opossite to "wisdom of crowds"
- Cascades easy to break (stop)

References

- A simple model of herd behavior, A Banerjee, The quarterly journal of economics, vol CVII, Issue 3, pp 797 -817, 1992
- A Theory of Fads, Fashion, Custom, and Cultural Change as Information Cascades, S. Bikhchandani, D Hirshleifer and I.Welch, 1992
- Information Cascades in the Laboratory, L. Anderson and C. Halt
- Following the Herd, Pierre Lemieux