

# Strategic Network Formation

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НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ  
УНИВЕРСИТЕТ

M. Jackson, A. Wolinsky, 1996

"A Strategic Model of Social and Economic Networks"

- why networks become the way they are
- people (agents) making rational choices establishing connections
- maximizing individual utility (incentives)
- connections bring costs and benefits
- stability of the network
- social efficiency (best for the society)
- friendship, professional, political, trade networks

- Distance-based utility

$$u_i(G) = \sum_{j \in N_i^{n-1}} \delta^{l_{ij}} - d_i \cdot c$$

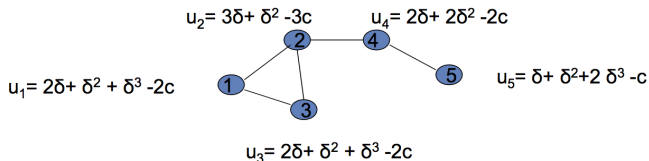
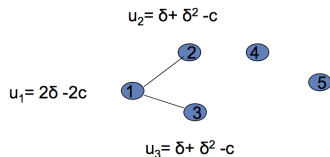
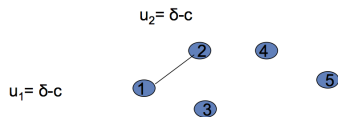
$l_{ij}$  - shortest path,  $d_i$  - node degree,  $\delta$ ,  $C$  - parameters,  $\delta < 1$

- Co-author model

$$u_i(G) = \sum_{j \in N_i} \left( \frac{1}{d_i} + \frac{1}{d_j} + \frac{1}{d_i d_j} \right), \quad d_i \neq 0$$

$u_i(G) = 1$  if  $d_i = 0$ ,  $d_i$  - node degree

# Distance-based utility function

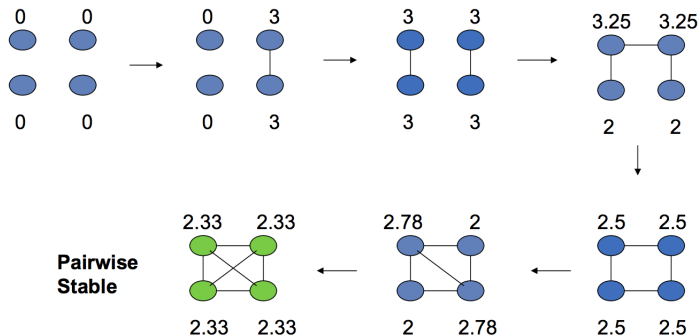


- Evolution: forming a link - mutual consent, removing a link - one person decision
- Network is pairwise stable if no player wants to remove a link and no two players want to add a link:

$$\forall i \quad u_i(G) \geq u_i(G - e_{ij})$$
$$\forall i, j \text{ if } u_i(G + e_{ij}) > u_i(G), \text{ then } u_j(G + e_{ij}) < u_j(G)$$

- Weak condition:
  - 1) considers removing one link at a time
  - 2) considers only forming one pair at a time

# Distance-based utility function



- Strong Efficiency, "best network", maximize total utility for the society

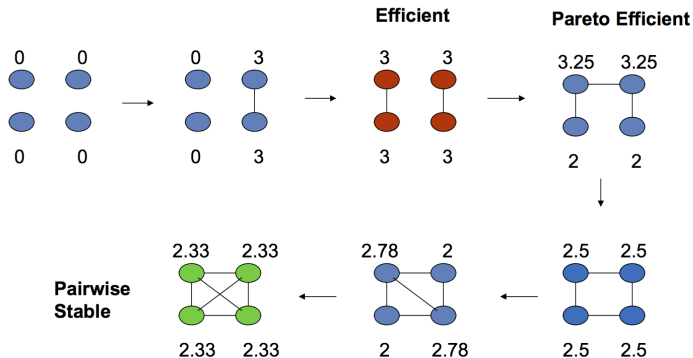
$$G^* = \max_G \sum_i u_i(G)$$

- Pareto Efficiency:

$$\nexists G' : \begin{aligned} u_i(G') &\geq u_i(G) \text{ for all } i \\ \text{and } u_i(G') &> u_i(G) \text{ for one } i \end{aligned}$$

- Efficiency  $\Rightarrow$  Pareto Efficiency

# Network Efficiency





- A Strategic Model of Social and Economic Networks, M. Jackson, A. Wolinsky, J. of Economic Theory, 71, pp44-74, 1996.