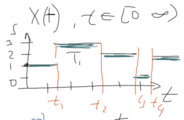
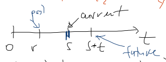
Lecture 5

$X \in E = \{0, 1, 2, \ldots, S\}$

$X(t)$, $t \in [0, \infty)$





$P(X(t+s)=j \mid X(s)=i \ \& \ X(r)=k) =$

$= P(X(t+s)=j \mid X(s)=i) =$

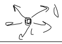
$= P(X(t)=j \mid X(0)=i) = p_{ij}(t)$

$p_{ij}(t) \leftrightarrow p_{ij}^{(n)}$

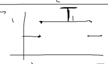
$p_{ij}(0) = \delta_{ij} = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases}$

$p_{ij}(t) \geq 0$

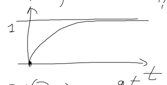
$\sum_{j=0}^{S} p_{ij}(t) = 1$, $\forall i$, $\forall t > 0$



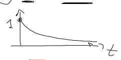
T_i :

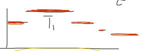


$P(T_i \leq t) = 1 - e^{-q_i t} = F_i(t)$



$P(T_i > t) = e^{-q_i t}$





$P(T_i > t+s \mid T_i > s) =$

$= \frac{P(T_i > t+s \ \& \ T_i > s)}{P(T_i > s)} =$

$= \frac{e^{-(t+s)q}}{e^{-sq}} = e^{-tq} = P(T_i > t)$

$P(T_1 > t+s \mid T_1 > t) = P(T_1 > s)$

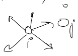
$= P(T_1 > s \mid T_1 > 0)$

$P(T_1 > t) = e^{-qt}$

$F(t) = P(T_i \leq t) = 1 - e^{-qt}$

$f(t) = F'(t) = q e^{-qt}$

$\langle T_i \rangle = E(t) = \int_0^{\infty} t f(t) dt = \frac{1}{q_i}$



$q_{ij} = \lim_{t \to 0} \frac{p_{ij}(t) - p_{ij}(0)}{t} = \frac{d p_{ij}(0)}{dt}$

$q_{ii} = \lim_{t \to 0} \frac{p_{ii}(t) - p_{ii}(0)}{t} =$, $p_{ii}(0) = 1$

$= \frac{d p_{ii}(0)}{dt}$

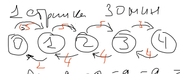
$q_i = \lim_{t \to 0} \frac{1 - p_{ii}(t)}{t} = -\frac{d p_{ii}(0)}{dt} = -q_{ii}$

$q_i = -q_{ii} = \frac{1}{\langle T_i \rangle}$

2 мастера / 2 кресла

1 клиент в 12 мин

1 стрижка 30 мин



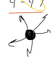
q_{ij} $q_{01} = q_{12} = q_{23} = q_{34} = 5$ (в час)

$q_{43} = 2 \cdot 2 = 4$ $q_{21} = 4$

$q_{32} = 4$ $q_{10} = 2$

$$Q = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} -5 & 5 & 0 & 0 & 0 \\ 2 & -7 & 5 & 0 & 0 \\ 0 & 4 & -9 & 5 & 0 \\ 0 & 0 & 4 & -9 & 5 \\ 0 & 0 & 0 & 4 & -4 \end{pmatrix} \end{array} \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

$q_{ii} = -q_i$



$\sum_j q_{ij} = 0$, $\forall i$

$\sum_j q_{ij} = \sum_{j \neq i} q_{ij} + q_{ii} = 0$

$q_{ii} = -q_i$

$q_i = \sum_{j \neq i} q_{ij}$

$p_{ij}(t) \leftarrow q_{ij} \leftarrow Q$

$Q = \begin{pmatrix} q_{00} & \cdots & q_{0S} \\ q_{S0} & \cdots & q_{SS} \end{pmatrix}$

T_i, $\langle T_i \rangle = \frac{1}{q_i}$