

lecture 7

DTMC  
CTMC = DTMC + exp. time e

$\lim_{h \rightarrow 0} P_{ij}^{(h)} = \delta_{ij}$ ,  $\sum_j P_{ij} = 1$

$\lim_{t \rightarrow 0} P_{ij}(t) = \delta_{ij}$

$\lim_{t \rightarrow \infty} \begin{pmatrix} p_{00}(t) & p_{01}(t) \\ p_{10}(t) & p_{11}(t) \end{pmatrix} = \begin{pmatrix} \frac{w_0}{w_0+w_1} & \frac{w_1}{w_0+w_1} \\ \frac{w_0}{w_0+w_1} & \frac{w_1}{w_0+w_1} \end{pmatrix}$

$\vec{w} P(t) = \vec{w}$   
 $\sum_i w_i p_{ij}(t) = w_j$

$\frac{dP_{ij}(t)}{dt} = \sum_{k \neq j} p_{ik}(t) p_{kj} - q_i p_{ij}(t)$

$P_{ij}(t) \rightarrow w_j$

$\frac{dP_{ij}(t)}{dt} \rightarrow 0$

$0 = \sum_{k \neq j} w_k p_{kj} - q_j w_j$

$w_j q_j = \sum_{k \neq j} w_k p_{kj}$

$\sum_j w_j = 1$

$q_{ij} = -q_i$   
 $\sum_{k \neq j} w_k p_{kj} = -w_j q_j$

$\sum_k w_k p_{kj} = 0$   
 $\sum_k w_k = 1$

$\vec{w} Q = 0$   
 $\vec{w} e = 1$

$Q = \begin{pmatrix} q_{00} & q_{01} \\ q_{10} & q_{11} \end{pmatrix}$

$\vec{w} \begin{pmatrix} q_{00} & q_{01} \\ q_{10} & q_{11} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\frac{dP(t)}{dt} = Q P(t)$

$\frac{dP(t)}{dt} = P(t) Q$

$\vec{w} = \vec{w} P(t)$

$\vec{w} = \vec{w} Q t^n$

$\vec{w} + \sum_{h>1} \vec{w} \frac{Q^h t^h}{h!}$

$\sum_{h>1} \vec{w} \frac{Q^h t^h}{h!} = 0$

$\vec{w} Q t + \vec{w} \frac{Q^2 t^2}{2!} + \vec{w} \frac{Q^3 t^3}{3!} = 0$

$\vec{w} Q = 0$

CTMC  $\rightarrow$  DTMC

$\Rightarrow \exists t p_{ij}(t) > 0 \Rightarrow i \rightarrow j$

Thompson's (1980)

$\vec{w} Q = 0$   
 $\vec{w} e = 1$

Birth & Death

$\{0, 1, \dots, M\}$   
 $q_{ij} = 0 \text{ if } |i-j| > 1$



M/M/1 Queue



$\lambda \geq \mu$