Special classes of networks

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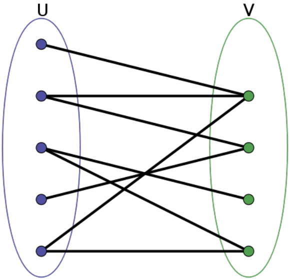
Bipartite graphs

Definition

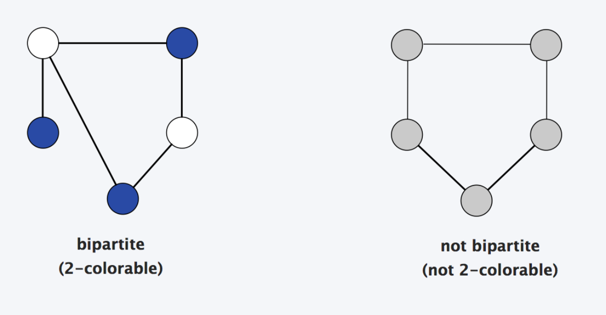
A bipartite graph (or bigraph, 2-mode network) is a graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in V.



- A bipartite graph does not contain any odd-length cycles
- A bipartite graph can be vertex colored wtih 2 colors

Testing bipartiteness

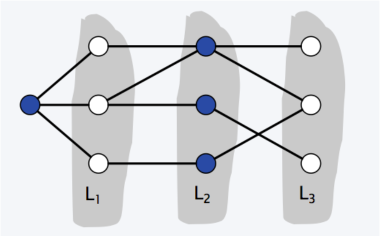
- Triangle is not bipartite (can't 2-color it)
- If graph contains an odd cycle it can't be bipartite



Testing bipartiteness

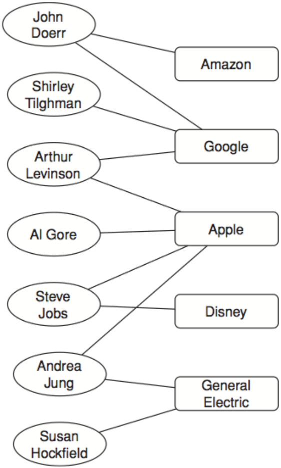
Is given graph a bipartite?

- Algorithm: Select and node and perform BFS, color each layer alternate colors
- Scan all the edges, see if any edge has nodes with the same color (one layer nodes)

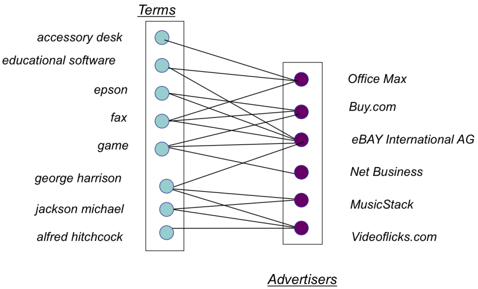


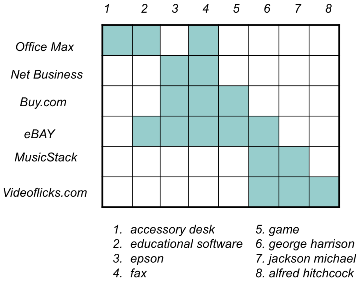
Affiliation networks

- two types of nodes
- can represent affiliation of people with foci
- people - groups
- candidates - jobs
- authors - papers
- directors - boards
- advertisers - keywords
- actors - movies
- people - raitings

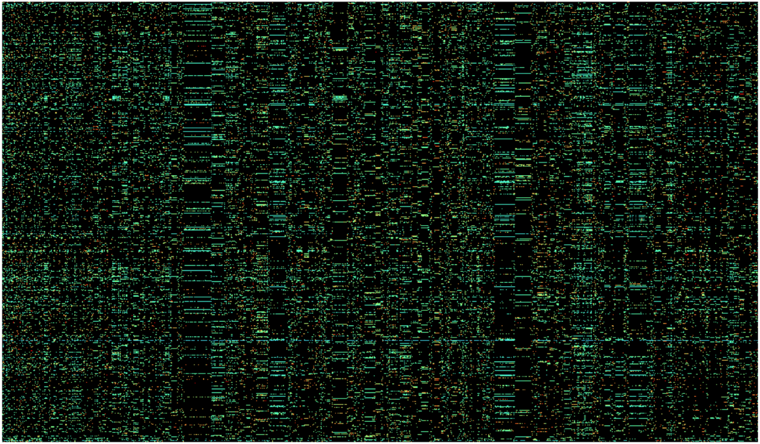


Sponsored search





Sponsored search



Incidence matrix

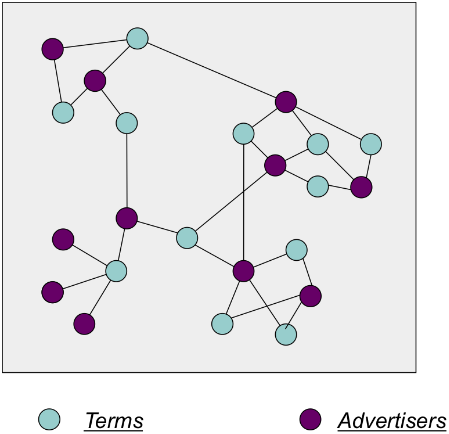
- m - users, n - groups
- Incidence matrix $B^{m \times n}$

$$B_{ij} = \begin{cases} 1: & \text{if user } i \text{ belongs to group } j \\ 0: & \text{otherwise} \end{cases}$$

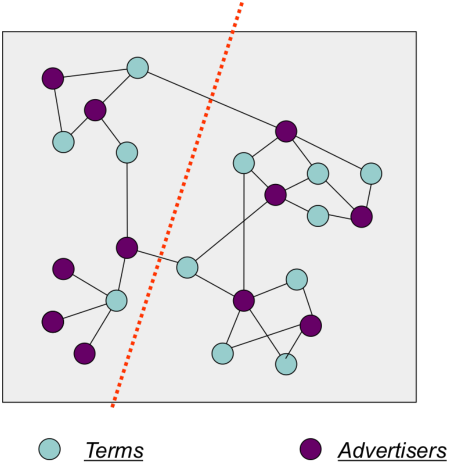
- $[1..m]$ nodes - users, $[m+1...m+n]$ nodes - groups
- Adjacency matrix $A^{(m+n)\times(m+n)}$

$$A = \begin{pmatrix} 0 & B \\ B^T & 0 \end{pmatrix}$$

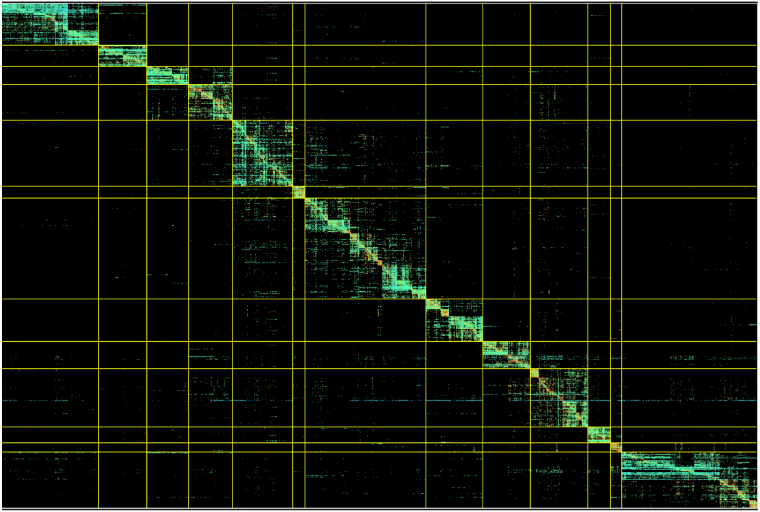
Bipartiate graph



Bipartiate graph partitioning



Sponsored search



One-mode projections

Two one-mode projections:

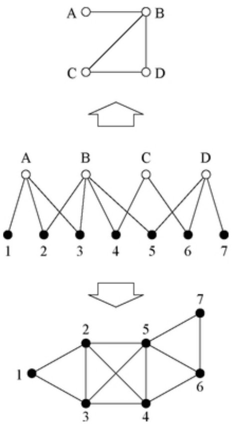
- Projection onto the users, user-user graph, $P'^{(m \times m)}$:

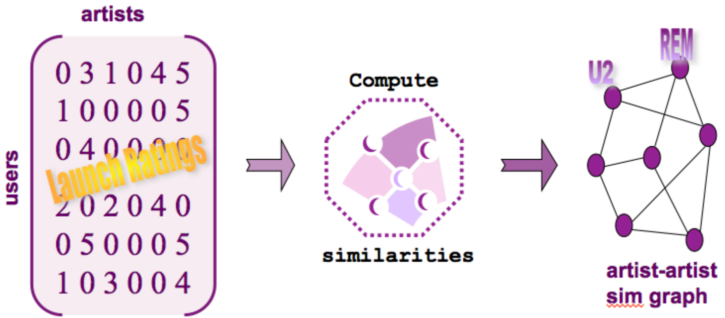
$$P' = BB^T$$

- Projection onto the groups, group-group graph, $P''^{(n \times n)}$:

$$P'' = B^T B$$

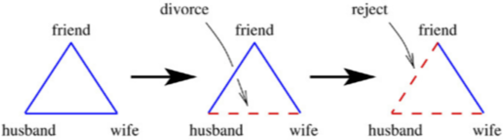
Both P' and P'' have self-loops




artists
users
0 3 1 0 4 5
1 0 0 0 0 5
Launch Ratings
2 0 2 0 4 0
0 5 0 0 0 5
1 0 3 0 0 4
Compute
similarities
U2
REM
artist-artist
sim graph

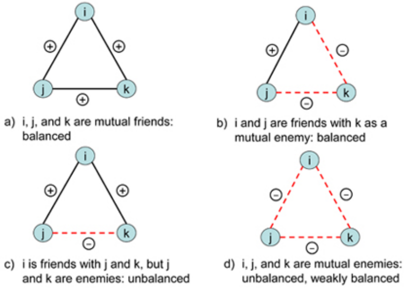
Networks with signed edges

- Social networks: friendship - positive edge, animosity - negative edge
- Local effect - global network properties
- Complete graphs (cliques)
- Dynamics of friendship, evolution of networks



Structural balance

- Social balance theory, Heider 1946, Cartwright and Harary, 1956
- Positivly connected nodes tend to match attitudes to third nodes
- Signed triangle, balanced state - algebraic multiplication sign is positive
- Complete graph is balances if every triange is balanced

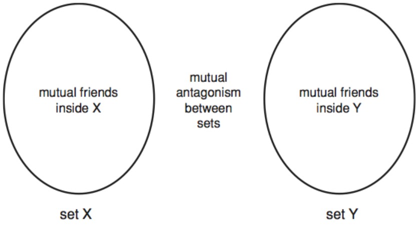


Cartwright et al, 1956

Balance Theorem

Theorem

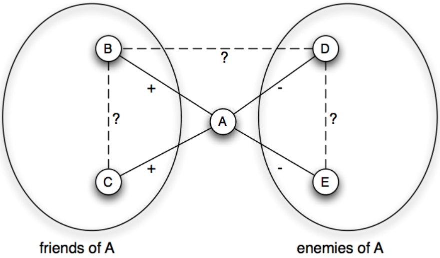
Balance Theorem: If a labeled complete graph is balanced, then either all pairs of nodes are friends, or else the nodes can be divided into two groups, X and Y , such that every pair of nodes in X like each other, every pair of nodes in Y like each other, and everyone in X is the enemy of everyone in Y.



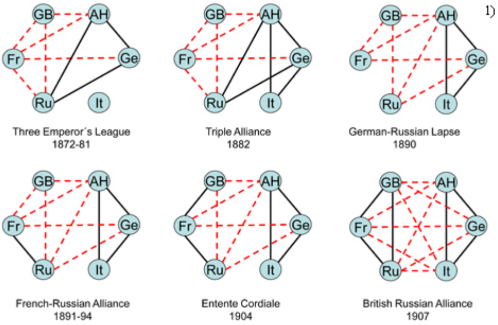
Balance Theorem

For balanced network (complete)

- Pick any node A
- Divide all nodes into friends of A (set X) and enemies of A (set Y)
- Any two nodes in X, (+)(+) with A, must be (+) between
- Any two nodes in X, (+)(+) with A, must be (+) between
- Any node from X and node from Y have (+)(-) with A, must be (-) between



International relations

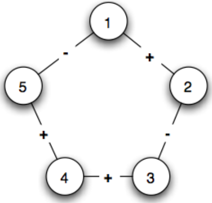


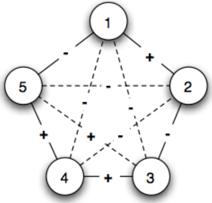
Antal et al, 2006

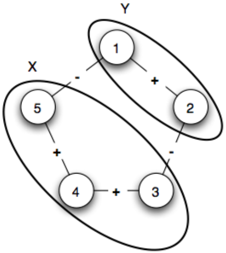
Structural balance in arbitraty networks

Non-complete network is balanced when:

- it is possible to fill missing labeled edges such that resulted complete graph is balanced
- divide network into mutually opposed sets of friends

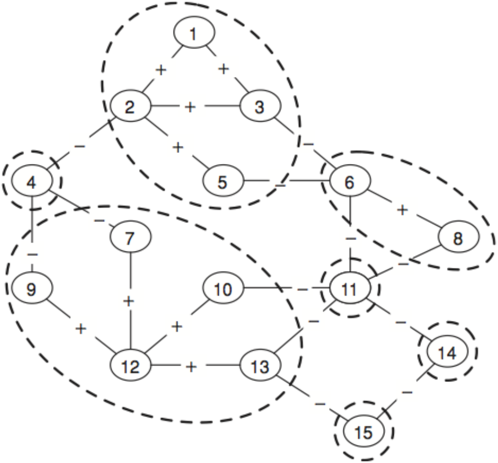






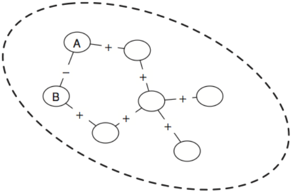
Structural balance

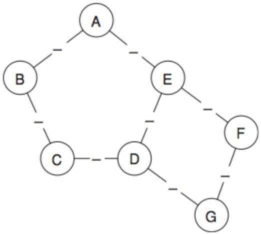
A signed graph is balanced if and only if it contains no cycle with an odd number of negative (-) edges



Structural balance

- Form supenodes from nodes connected by positive edges
- Internal supernode consistency: if there is a negative edge, it contains a loop with single negative (odd number)
- Can assign labels X and Y to each supernode
- Consitency of reduced graph - 2 way coloring problem!
- Bipartite graph - no odd loops allowed





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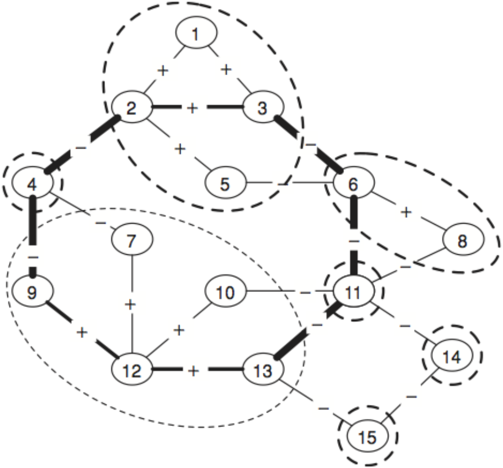

References

- Chapter 5. David Easley and John Kleinberg."Networks, Crowds, and Markets: Reasoning About a Highly Connected World."Cambridge University Press 2010.
- Leonid Zhukov. Spectral Clustering of Large Advertiser Datasets. Overture R&D Technical Report, 2003.
- D. Cartwright, F. Harary. Structural balance: A generalization of Heider's theory. Psychological review, 1956.

Module 3 Review

- Network properties
  - Node degree distribution
  - Small diameter
  - Clustering coefficient
- Network models
  - Random graph
  - Small world
  - Preferential attachement

Module 3 Review

- Node metrics
  - Degree centrality
  - Closeness centrality
  - Betweenness centrality
  - Eigenvector centrality
  - Katz centrality
- Link analysis
  - Pagerank
  - Hubs and Authorities
- Node similarity
  - Structural equivalence
  - Similarity matrix
  - Assortative mixing

Module 3 Review

- Community detection
  - Modlarity optimization
  - Edge betweenness
  - Randomized min cut
  - Multilevel graph partitioning
  - Local clustering /random walks
- Network structure
  - Graph k-cores
  - Network motifs
- Special networks
  - Affiliation networks
  - Signed networks