Social contagion

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Social contagion

- **Diffusion of innovation - Threshold model:**
  - Adoption of innovations
  - Joining a protest group
  - Making a purchase (?)

- **Social diffusion - Independent cascade model:**
  - News propagation
  - Retweets, mentions, email forwarding
  - Recommendations
Influence response: diminishing returns and critical mass (threshold)

\[ P(n) = 1 - (1 - p)^n \quad \text{and} \quad P(b) = \delta(b > b_0) \]
Threshold model

Network coordination game, direct-benefit effect

Node $v$ to make decision $A$ or $B$, $p$ - portion of type $A$ neighbors to accept $A$:

$$a \cdot p \cdot d > b \cdot (1 - p) \cdot d$$

$$p \geq b/(a + b)$$

Threshold:

$$q = \frac{b}{a + b}$$

Accept new behavior $A$ when $p \geq q$
Cascade propagation

Cascade - sequence of changes of behavior, "chain reaction"

- Let $a = 3$, $b = 2$, threshold $q = \frac{2}{(2 + 3)} = \frac{2}{5}$
- Start from nodes 7,8: $\frac{1}{3} < \frac{2}{5} < \frac{2}{3}$
- Cascade size - number of nodes that changed the behavior
- Complete cascade when every node changes the behavior
Cascades and clusters

Group of nodes form a cluster of density $\rho$ if every node in the set has at least fraction $\rho$ of its neighbors in the set.

If the network contains a cluster with density $\rho > 1 - q$, then the cluster will block the propagation of cascade and there will be no complete cascade.

For cascade to get into cluster $q \leq 1 - \rho$.

Adoption threshold to get into the left cluster $q = 2/5$, the right cluster $q = 1/3$. 
Cascades capacity

- Cascade capacity - the largest threshold $q_\infty = \max b/(a + b)$ at which small set of initial adopters can cause a complete cascade?
- Finite set of initial nodes on an infinite network

![Diagram showing network with initial adopters highlighted]

$q_\infty = 1/2$

$q_\infty = 3/8$

- With smaller $q < q_\infty$ cascade spreads faster
- For $q < 1/2$, $a > b$; for $q < 3/8$, $a > 5/3b$
Cascade capacity

- The bigger the cascade capacity $q_\infty$, the easier cascade spreads on network
- If $q_\infty > 1/2$ exists, then $a < b$ can cause a complete cascade, i.e. inferior technology could displace superior with a small set of initial adopters
- Claim: There is no network with cascade capacity $q_\infty > 1/2$

![Diagram](image)

Shrinking interface: since $p > q_\infty > 1/2$, there are more connections inside interface then outside on every step, moving in decreases interface size
Ryan-Gross study of hybrid seed corn delayed adoption (after first exposure)

Ryan and Gross, 1943
Independent cascade model

- Initial set of active nodes $S_0$
- Discrete time steps
- On every step an active node $v$ can activate connected neighbour $w$ with a probability $p_{v,w}$ (single chance)
- If $v$ succeeds, $w$ becomes active on the next time step
- Process runs until no more activations possible
10 mln members, joining groups

Backstrom, 2006
Online recommendation network

- Large on-line retailer. After purchase one could send recommendation to a friend, both receive discount/referral credit
- 15 mln recommendations, 3 mln users

Leskovec et.al, 2006
Online recommendation network

(a) Medical book

(b) Japanese graphic novel

Leskovec et.al, 2006
Online recommendation network

Most frequent subgraphs in cascade (network motifs)

Leskovec et.al, 2006
Online newsletter experiment

- Online email newsletter + reward for recommending it
- 7,000 seed, 30,000 total recipients, around 7,000 cascades, size form 2 nodes to 146 nodes, max depth 8 steps
- on average $k = 2.96$, infection probability $p = 0.00879$
- Suprisingly: no loops, triangles, closed paths

Iribarren et.al, 2009
Cascade - branching process
Branching process:

- each node has $k$ neighbours, transmits "infection" with probability $p$
- let $q_n$ - probability that infection persists $n$ steps (levels of the tree)
- $pq_{n-1}$ - probability that spreads through first contact and then survives $n - 1$ levels
- $(1 - pq_{n-1})^k$ - probability that will not spread through any of the nodes

$$1 - q_n = (1 - pq_{n-1})^k$$
Branching process

- limiting probability \( q^* = \lim_{n \to \infty} q_n \)
  
  \[
  q^* = 1 - (1 - pq^*)^k
  \]

- Slope: \( pk(1 - pq)^{k-1} \), at \( q = 0 \), \( R_0 = pk \) - reproductive number

- When \( R_0 > 1 \), there is a non zero probability of infection persists

- For newsletter email campaign: \( R_0 = 0.00879 \times 2.96 \approx 0.26 < 1 \)
Two main models

- **Independent Cascade Model** - diminishing returns (epidemic model):
  - spread quickly in highly connected networks
  - long range links benefit spreading
  - high degree nodes (hubs) help spreading

- **Linear Threshold model** - critical mass (game-theoretic model):
  - spread quickly in locally connected networks (inside clusters)
  - outside clusters slows the spreading
  - high degree nodes slow spreading
References

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- The Dynamics of Viral Marketing, J. Leskovec, L. Adamic, B. Huberman, 2006