Information cascades

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Aggregation of information

- Wisdom of crowd - taking into account collective opinion of a group of individuals for collective decision:
  - private information
  - independent judgements
  - aggregation process

- "Vox populi" - "The voice of the people", Francis Galton, Nature, 1907

- Claim: collective decision is better than decision by any individual

- Rational bubbles? - herd behaviour producing very bad group judgment, "madness of crowds", irrationality

- We do not consider psychological effects - conformity from social (peer) pressure, etc

- Rational behaviour, but a systemic flaw in information aggregation process

"The Wisdom of Crowds", James Surowiecki, 2005
Observational learning

Observational learning - influence resulting from rational processing of information gained by observing others

- Observational learning can be one of the causes of *convergent behaviour*
- Spread of fashion, fads, music hits, technology adoptions, financial markets, riots, etc.
- Make decision after observing the past decisions of others by making inferences
- Observable actions, but not observable reasons (private signals)
- There is no "true" learning, behavior is imitative
- Rational decision making

"A simple model of herd behavior", Abhijit Banerjee, 1992
Information cascade

Definition

Information cascade occurs when individuals (agents), having observed the actions of those ahead of them, rationally choose to take the same action regardless of their private information.

- Agents make decision sequentially.
- Every agent observes decisions of others before making own decision.
- Every agent make rational decision based on the information he has (his private information + observed previous decisions).
- Agents do not have access to private information of others (only decisions they made).
- Only limited action (decision) space exists.
Baysian learning

- Hypothesis testing: $H_1, H_2$
- Apriory probability: $P(H_1), P(H_2)$
- Observed evidence: $E$
- Aposteriory: $P(H_1|E), P(H_2|E)$
- Bayes’s rule:

$$P(H_1|E) = \frac{P(E|H_1)P(H_1)}{P(E)}$$

$$P(H_2|E) = \frac{P(E|H_2)P(H_2)}{P(E)}$$

$$P(E) = P(E|H_1)P(H_1) + P(E|H_2)P(H_2)$$
Experiment: sequential marble drawing from a random urn

Urn A, B:
- \( P(A) = P(B) = \frac{1}{2} \)

Marbles a, b:
- \( P(a|A) = \frac{2}{3}, \ P(b|A) = \frac{1}{3} \)
- \( P(a|B) = \frac{1}{3}, \ P(b|B) = \frac{2}{3} \)

L. Anderson and C. Halt, 1997
Simple experiment

- Step 1. Selected "b-marble", $P(B|b) \neq P(A|b)$

\[
P(B|b) = \frac{P(b|B)P(B)}{P(b)}
\]

\[
P(b) = P(b|B)P(B) + P(b|A)P(A) = 2/3 \cdot 1/2 + 1/3 \cdot 1/2 = 1/2
\]

\[
P(B|b) = \frac{2/3 \cdot 1/2}{1/2} = 2/3
\]

\[
P(A|b) = \frac{1/3 \cdot 1/2}{1/2} = 1/3
\]

- Rational choice announce "B -urn" : $P(B|b) > P(A|b)$
- Exposes private signal $b$
Simple experiment

Step 2 (a). Selected "b-marble", $P(B|B, b) ? P(A|B, b)$

$$P(B|B, b) = P(B|b, b) = \frac{P(b, b|B)P(B)}{P(b, b)}$$

$$P(b, b|B) = P(b|B)P(b|B) = 2/3 \cdot 2/3 = 4/9$$

$$P(b, b|A) = P(b|A)P(b|A) = 1/3 \cdot 1/3 = 1/9$$

$$P(b, b) = P(b, b|B)P(B) + P(b, b|A)P(A) =$$

$$= 4/9 \cdot 1/2 + 1/9 \cdot 1/2 = 5/18$$

$$P(B|b, b) = \frac{4/9 \cdot 1/2}{5/18} = 4/5$$

$$P(A|b, b) = \frac{1/9 \cdot 1/2}{5/18} = 1/5$$

Rational choice announce "B-urn" : $P(B|B, b) > P(A|B, b)$

Exposes private signal $b$
Step 2(b). Selected “a-marble”, \( P(B|B, a) \) ? \( P(A|B, a) \)

\[
P(B|b, a) = \frac{P(b, a|B)P(B)}{P(b, a)}
\]

\[
P(b, a|B) = P(b|B)P(a|B) = 2/3 \cdot 1/3 = 2/9
\]

\[
P(b, a|A) = P(b|A)P(b|A) = 1/3 \cdot 2/3 = 2/9
\]

\[
P(b, a) = P(b, a|B)P(B) + P(b, a|A)P(A) =
\]

\[
= 2/9 \cdot 1/2 + 2/9 \cdot 1/2 = 2/9
\]

\[
P(B|b, a) = \frac{2/9 \cdot 1/2}{2/9} = 1/2
\]

\[
B(A|b, a) = \frac{2/9 \cdot 1/2}{2/9} = 1/2
\]

- Rational choice to follow own signal, announce "A-urn"
- Exposes private signal \( b \)
Step 3. Selected "a-marble" $P(B|B, B, a) > P(A|B, B, a)$ inspite of own signal!

Action does not expose any private signal
**Table 2—Data for Selected Periods of Session 2**

<table>
<thead>
<tr>
<th>Period</th>
<th>Urn used</th>
<th>1st round</th>
<th>2nd round</th>
<th>3rd round</th>
<th>4th round</th>
<th>5th round</th>
<th>6th round</th>
<th>Cascade outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>B</td>
<td>S12: A (a)</td>
<td>S11: B (b)</td>
<td>S9: B (b)</td>
<td>S7: B (b)</td>
<td>S8: B (a)</td>
<td>S10: B (a)</td>
<td>cascade</td>
</tr>
<tr>
<td>6</td>
<td>A</td>
<td>S12: A (a)</td>
<td>S8: A (a)</td>
<td>S9: A (b)</td>
<td>S11: A (b)</td>
<td>S10: A (a)</td>
<td>S7: A (a)</td>
<td>cascade</td>
</tr>
<tr>
<td>7</td>
<td>B</td>
<td>S8: B (b)</td>
<td>S7: A (a)</td>
<td>S10: B (b)</td>
<td>S11: B (b)</td>
<td>S12: B (a)</td>
<td>S9: B (a)</td>
<td>cascade</td>
</tr>
<tr>
<td>8</td>
<td>A</td>
<td>S8: A (a)</td>
<td>S9: A (a)</td>
<td>S12: B* (b)</td>
<td>S10: A (a)</td>
<td>S11: A (b)</td>
<td>S7: A (a)</td>
<td>cascade</td>
</tr>
<tr>
<td>9</td>
<td>B</td>
<td>S11: A (a)</td>
<td>S12: A (a)</td>
<td>S8: A (b)</td>
<td>S9: A (b)</td>
<td>S7: A (b)</td>
<td>S10: A (b)</td>
<td>reverse cascade</td>
</tr>
</tbody>
</table>

*Notes: Boldface — Bayesian decision, inconsistent with private information.
*— Decision based on private information, inconsistent with Bayesian updating.

L. Anderson and C. Halt, 1997
General Cascade Model

- Group of agents \( \{1, \ldots, n\} \) sequentially making decisions accepting/rejecting an option.

- State of the world (one of two possible, random):
  - 'G' - good, 'B' - bad,
  - \( Pr[G] = p, \ Pr[B] = 1 - p \)

- Payoff: \( v_G > 0, \ v_B < 0 \)
  Expected payoff without any information \( v_G p + v_B (1 - p) = 0 \)

- Private signal:
  - 'H' - accepting is a good idea, 'L' - accepting is a bad idea.
  Random, but truthful, \( q > 1/2, \ q \) - signal accuracy
  - \( Pr[H|G] = q, \ Pr[L|G] = 1 - q \)
  - \( Pr[H|B] = 1 - q, \ Pr[L|B] = q \)
General Cascade Model

- No signal:

\[ E^{\text{no-signal}}[\text{payoff}] = v_G \Pr[G] + v_B \Pr[B] = v_G p + v_B (1 - p) = 0 \]

- Individual decisions: High signal 'H':

\[
Pr[G|H] = \frac{Pr[H|G]Pr[G]}{Pr[H]} = \frac{qp}{qp + (1-q)(1-p)} > p
\]
\[
Pr[B|H] = \frac{Pr[H|B]Pr[B]}{Pr[H]} = \frac{(1-q)(1-p)}{qp + (1-q)(1-p)} < 1 - p
\]

\[ E^{\text{signal}}[\text{payoff}] = v_G \Pr[G|H] + v_B \Pr[B|H] > E^{\text{no-signal}}[\text{payoff}] \]

- Rational agent should accept the option
Mutliple signals $S = \{HLH..LHLL\}$, $a = \#H,b = \#L$

Posterior probability:

$$Pr[G|S] = \frac{Pr[S|G]Pr[G]}{Pr[S]} = \frac{pq^a(1-q)^b}{pq^a(1-q)^b + (1-p)(1-q)^a q^b}$$

if $a > b$, $Pr[G|S] > Pr[G]$
if $a < b$, $Pr[G|S] < Pr[G]$
if $a = b$, $Pr[G|S] = Pr[G] = p$

Rational agent should accept the option when gets more H signals than L
Sequential decision making

Each person can see the choice of previous people, but not their signals

1. Person 1. Follow private signal (1). Action reveals his private signal

2. Person 2. Follows 2 signals = his private (2) + private signal (1)
   if private (2) = private (1), follows his private signal (1)
   if private (2) ≠ private(1), follows his private
   Action reveals his private signal (2)

3. Person 3. Follows 3 signals = his private (3) + private (2) + private(1)
   if private (1) ≠ private (2), follows his private signal (3)
   if private (1) = private (2), follows signals (1), (2), not his private
   signal (3)
   Action *does not* reveal his private signal

4. Person 4 etc. If private (1) = private (2), follows signals (1), (2), not
   his private signal (4)
Information cascade

- when number of previous accepts = rejects, follows own signal
- when number of previous \(| \text{accepts} - \text{rejects} | = 1 \), follows own signal
- cascades start when \(| \text{accepts} - \text{rejects} | \geq 2 \), private signal can’t outweigh earlier majority
Information cascades

Source: Hirshleifer (1995)
Information cascades

Let the true state of the world be ‘G’. Probability of cascade after 2 people

- Probability of Up (correct) cascade:
  \[ \Pr[HH] = q^2, \]
  \[ \Pr[HL] = q(1 - q) \]
  \[ \Pr[Up \text{ cascade}] = q^2 + q(1 - q)1/2 = q(q + 1)/2 \]
  \[ q = 0.5 \Rightarrow \Pr = 37.5\%, \quad q = 0.6 \Rightarrow \Pr = 48\%, \]

- Probability of No cascade:
  \[ \Pr[HL] = q(1 - q), \]
  \[ \Pr[LH] = q(1 - q) \]
  \[ \Pr[No \text{ cascade}] = q(1 - q)1/2 + q(1 - q)1/2 = q(1 - q) \]
  \[ q = 0.5 \Rightarrow \Pr = 25\%, \quad q = 0.6 \Rightarrow \Pr = 24\%, \]

- Probability of Down (incorrect) cascade:
  \[ \Pr[LL] = (1 - q)^2, \]
  \[ \Pr[LH] = q(1 - q) \]
  \[ \Pr[Down \text{ cascade}] = (1 - q)^2 + q(1 - q)1/2 = (1 - q)(2 - q)/2 \]
  \[ q = 0.5 \Rightarrow \Pr = 37.5\%, \quad q = 0.6 \Rightarrow \Pr = 28\%, \]
Information cascades

Probability of cascade after $n$ (even) people

- $Pr[No\ cascade] = (q - q^2)^{n/2}$
- $Pr[Up\ cascade] = \frac{q(q+1)(1-(q-q^2)^{n/2})}{2(1-q-q^2)}$ - "correct" cascade
- $Pr[Down\ cascade] = \frac{(q-2)(q-1)(1-(q-q^2)^{n/2})}{2(1-q-q^2)}$ - "incorrect" cascade
Cascades start when agents have incomplete information and observe actions of others.

Cascades very easy to start (2 +).

With large number of people a cascade happens almost surely

\[ \lim_{n \to \infty} (q - q^2)^{n/2} \to 0 \]

Cascades prevent information aggregation ("wisdom of crowd"), start based on little information.

Cascades can be wrong - incorrect cascade.

Cascades easy to break (stop).

Very important early actions/actors in cascades.

Extensions: don’t see all the previous decisions, various strength of private signals, different payoff etc.
References

- Information Cascades in the Laboratory, L. Anderson and C. Halt
- Following the Herd, Pierre Lemieux