Information cascades

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Aggregation of information

- Wisdom of crowd - taking into account collective opinion of a group of individuals for collective decision:
  - private information
  - independent judgements
  - aggregation process
- "Vox populi" - "The voice of the people", Francis Galton, Nature,1907
- Claim: collective decision is better that decision by any individual
- Rational bubles? - herd behaviour producing very bad group judgment, "madness of crowds", irrationality
- We do not consider psychological effects - conformity from social (peer) pressure, etc
- Rational behaviour, but a systemic flaw in information aggregation process

"The Wisdom of Crowds", James Surowiecki, 2005

Observational learning

Observational learning - influence resulting from rational processing of information gained by observing others

- Observational learning can be one of the causes of *convergent behaviour*
- Spread of fashion, fads, music hits, techonology adoptions, financial markets, riots, etc.
- Make decision after observing the past decisions of others by making inferences
- Observable actions, but not observable reasons (private signals)
- There is no "true" learning, behavior is imitative
- Rational decision making

"A Theory of Fads, Fashion, Custom, and Cultural Change as Information Cascades", S. Bikhchandani, D Hirshleifer and I.Welch, 1992
"A simple model of herd behavior", Abhijit Banerjee, 1992

Information cascade

Definition

Information cascade occurs when individuals (agents), having observed the actions of those ahead of them, rationally choose to take the same action regardless of their private information

- Agents make decision sequentially
- Every agent observes decisons of others before making own decision
- Every agent make rational decision based on the information he has (his private information + observed previous decisons)
- Agents do not have access to private information of others (only decisions they made)
- Only limited action (decision) space exists

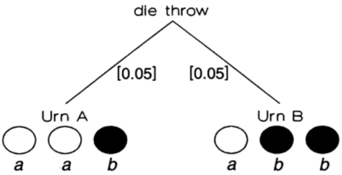
Baysian learning

- Hypothesis testing: H_1, H_2
- Apriory probability: $P(H_1)$, $P(H_2)$
- Observed evidence: E
- Aposteriory: $P(H_1|E)$, $P(H_2|E)$
- Bayes's rule:

$$\begin{aligned}
P(H_1|E) &= \frac{P(E|H_1)P(H_1)}{P(E)} \\
P(H_2|E) &= \frac{P(E|H_2)P(H_2)}{P(E)} \\
P(E) &= P(E|H_1)P(H_1) + P(E|H_2)P(H_2)
\end{aligned}$$

Simlpe experiment

Experiment: sequential marble drawing from a random urn



- Urns A, B:
  $P(A) = P(B) = 1/2$
- Marbles a,b:
  $P(a|A) = 2/3,\ P(b|A) = 1/3$
  $P(a|B) = 1/3,\ P(b|B) = 2/3$

L. Anderson and C. Halt, 1997

Simlpe experiment

- Step 1. Selected "b-marble", $P(B|b)$? $P(A|b)$

$$
\begin{aligned}
P(B|b) &= \frac{P(b|B)P(B)}{P(b)} \\
P(b) &= P(b|B)P(B) + P(b|A)P(A) = \\
&= 2/3 \cdot 1/2 + 1/3 \cdot 1/2 = 1/2 \\
P(B|b) &= \frac{2/3 \cdot 1/2}{1/2} = 2/3 \\
P(A|b) &= \frac{1/3 \cdot 1/2}{1/2} = 1/3
\end{aligned}
$$

- Rational choice announce "B -urn" : $P(B|b) > P(A|b)$
- Exposes private signal b

Simlpe experiment

- Step 2 (a). Selected "b-marble", $P(B|B,b)$? $P(A|B,b)$

$$
\begin{aligned}
P(B|B,b) = P(B|b,b) &= \frac{P(b,b|B)P(B)}{P(b,b)} \\
P(b,b|B) &= P(b|B)P(b|B) = 2/3 \cdot 2/3 = 4/9 \\
P(b,b|A) &= P(b|A)P(b|A) = 1/3 \cdot 1/3 = 1/9 \\
P(b,b) &= P(b,b|B)P(B) + P(b,b|A)P(A) = \\
&= 4/9 \cdot 1/2 + 1/9 \cdot 1/2 = 5/18 \\
P(B|b,b) &= \frac{4/9 \cdot 1/2}{5/18} = 4/5 \\
P(A|b,b) &= \frac{1/9 \cdot 1/2}{5/18} = 1/5
\end{aligned}
$$

- Rational choice announce "B-urn" : $P(B|B,b) > P(A|B,b)$
- Exposes private signal b

Simlpe experiment

- Step 2(b). Selected "a-marble", $P(B|B,a)$? $P(A|B,a)$

$$
\begin{aligned}
P(B|b,a) &= \frac{P(b,a|B)P(B)}{P(b,a)} \\
P(b,a|B) &= P(b|B)P(a|B) = 2/3 \cdot 1/3 = 2/9 \\
P(b,a|A) &= P(b|A)P(b|A) = 1/3 \cdot 2/3 = 2/9 \\
P(b,a) &= P(b,a|B)P(B) + P(b,a|A)P(A) = \\
&= 2/9 \cdot 1/2 + 2/9 \cdot 1/2 = 2/9 \\
P(B|b,a) &= \frac{2/9 \cdot 1/2}{2/9} = 1/2 \\
B(A|b,a) &= \frac{2/9 \cdot 1/2}{2/9} = 1/2
\end{aligned}
$$

- Rational choice to follow own signal, announce "A-urn"
- Exposes private signal b

Simlpe experiment

- Step 3. Selected "a-marble" $P(B|B,B,a)$? $P(A|B,B,a)$

$$
\begin{aligned}
P(B|b,b,a) &= \frac{P(b,b,a|B)P(B)}{P(b,b,a)} \\
P(b,b,a|B) &= P(b|B)P(b|B)P(a|B) = 2/3 \cdot 2/3 \cdot 1/3 = 4/27 \\
P(b,b,a|A) &= P(b|A)P(b|A)P(a|A) = 1/3 \cdot 1/3 \cdot 2/3 = 2/27 \\
P(b,b,a) &= P(b,b,a|B)P(B) + P(b,b,a|A)P(A) = \\
&= 4/27 \cdot 1/2 + 2/27 \cdot 1/2 = 3/27 \\
P(B|b,b,a) &= \frac{4/27 \cdot 1/2}{3/27} = 2/3 \\
P(A|b,b,a) &= \frac{2/27 \cdot 1/2}{3/27} = 1/3
\end{aligned}
$$

- Rational choice "B-urn" : $P(B|B,B,a) > P(A|B,B,a)$ inspite of own signal!
- Action does not expose any private signal

Simple experiment

TABLE 2—DATA FOR SELECTED PERIODS OF SESSION 2

		Subject number: Urn decision (private draw)						
Period	Urn used	1st round	2nd round	3rd round	4th round	5th round	6th round	Cascade outcome
5	B	S12: A (*a*)	S11: B (*b*)	S9: B (*b*)	S7: B (*b*)	**S8: B (*a*)**	**S10: B (*a*)**	cascade
6	A	S12: A (*a*)	S8: A (*a*)	**S9: A (*b*)**	**S11: A (*b*)**	S10: A (*a*)	S7: A (*a*)	cascade
7	B	S8: B (*b*)	S7: A (*a*)	S10: B (*b*)	S11: B (*b*)	S12: B (*b*)	**S9: B (*a*)**	cascade
8	A	S8: A (*a*)	S9: A (*a*)	S12: B* (*b*)	S10: A (*a*)	**S11: A (*b*)**	S7: A (*a*)	cascade
9	B	S11: A (*a*)	S12: A (*a*)	**S8: A (*b*)**	**S9: A (*b*)**	**S7: A (*b*)**	**S10: A (*b*)**	reverse cascade

Notes: Boldface—Bayesian decision, inconsistent with private information.
*—Decision based on private information, inconsistent with Bayesian updating.

L. Anderson and C. Halt, 1997

General Cascade Model

- Group of agents $\{1, .., n\}$ sequentially making decisions accepting/rejecting an option
- State of the world (one of two possible, random):
  'G' - good, 'B' - bad,
  $Pr[G] = p$, $Pr[B] = 1 - p$
- Payoff: $v_G > 0$, $v_B < 0$
  Expected payoff without any information $v_G p + v_B(1 - p) = 0$
- Private signal:
  'H' - accpeting is a good idea, 'L' - acceepting is a bad idea.
  Random, but truthful, $q > 1/2$, q - signal accuracy
  $Pr[H|G] = q$, $Pr[L|G] = 1 - q$
  $Pr[H|B] = 1 - q$, $Pr[L|B] = q$

General Cascade Model

- No signal:

$$E^{no-signal}[payoff] = v_G Pr[G] + v_B Pr[B] = v_G p + v_B(1-p) = 0$$

- Individual decisions: High signal 'H':

$$\begin{aligned} Pr[G|H] &= \frac{Pr[H|G]Pr[G]}{Pr[H]} = \frac{qp}{qp+(1-q)(1-p)} > p \\ Pr[B|H] &= \frac{Pr[H|B]Pr[B]}{Pr[H]} = \frac{(1-q)(1-p)}{qp+(1-q)(1-p)} < 1-p \end{aligned}$$

$$E^{signal}[payoff] = v_G Pr[G|H] + v_B Pr[B|H] > E^{no-signal}[payoff]$$

- Rational agent should accept the option

General Cascade Model

- Mutliple signals $S = \{HLH..LHLL\}$, $a = \#H, b = \#L$
- Posterior probability:

$$Pr[G|S] = \frac{Pr[S|G]Pr[G]}{Pr[S]} = \frac{pq^a(1-q)^b}{pq^a(1-q)^b + (1-p)(1-q)^a q^b}$$

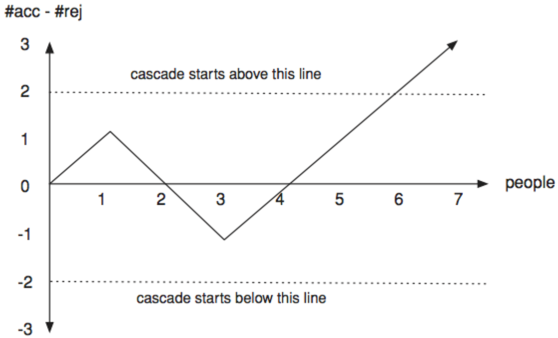
- if $a > b$, $Pr[G|S] > Pr[G]$
  if $a < b$, $Pr[G|S] < Pr[G]$
  if $a = b$, $Pr[G|S] = Pr[G] = p$
- Rational agent should accept the option when gets more H signals than L

Sequential decision making

Each person can see the choice of previous people, but not their signals

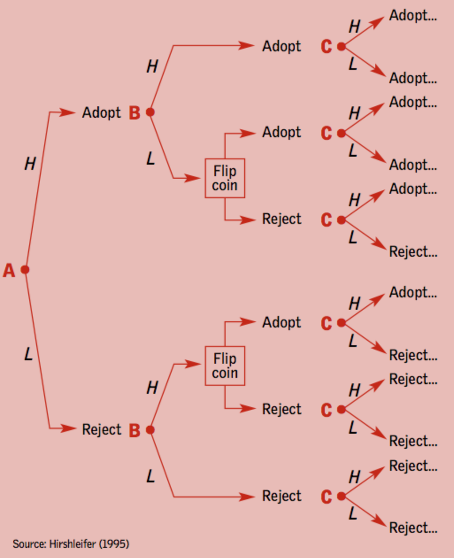
1. Person 1. Follow private signal (1). Action reveals his private signal
2. Person 2. Follows 2 signals = his private (2) + private signal (1)
   if private (2) = private (1), follows his private signal (1)
   if private (2) $\neq$ private(1), follows his private
   Action reveals his private signal (2)
3. Person 3. Follows 3 signals = his private (3) + private (2) + private(1)
   if private (1) $\neq$ private (2), follows his private signal (3)
   if private (1) = private (2), follows signals (1), (2), not his private signal (3)
   Action *does not* reveal his private signal
4. Person 4 etc. If private (1) = private (2), follows signals (1), (2), not his private signal (4)

Information cascade



- when number of previous accepts = rejects, follows own sinal
- when number of previous |accepts - rejects | = 1, folows own signal
- cascades starts when | accepts -rejects| $\geq$ 2, private signal can't outweight earlier majority

Information cascades



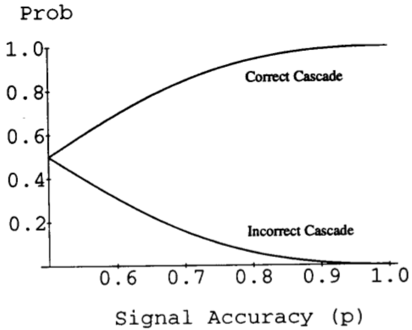
Information cascades

Let the true state of the world be 'G'. Probability of cascade after 2 people

- Probability of Up (correct) cascade:
  $Pr[HH] = q^2$,
  $Pr[HL] = q(1-q)$
  $Pr[Up\ cascade] = q^2 + q(1-q)1/2 = q(q+1)/2$
  $q = 0.5 \Rightarrow Pr = 37.5\%$, $q = 0.6 \Rightarrow Pr = 48\%$,
- Probability of No cascade:
  $Pr[HL] = q(1-q)$,
  $Pr[LH] = q(1-q)$
  $Pr[No\ cascade] = q(1-q)1/2 + q(1-q)1/2 = q(1-q)$
  $q = 0.5 \Rightarrow Pr = 25\%$, $q = 0.6 \Rightarrow Pr = 24\%$,
- Probability of Down (incorrect) cascade:
  $Pr[LL] = (1-q)^2$,
  $Pr[LH] = q(1-q)$
  $Pr[Down\ cascade] = (1-q)^2 + q(1-q)1/2 = (1-q)(2-q)/2$
  $q = 0.5 \Rightarrow Pr = 37.5\%$, $q = 0.6 \Rightarrow Pr = 28\%$,

Probability of cascade after n (even) people

- $Pr[No\ cascade] = (q - q^2)^{n/2}$
- $Pr[Up\ cascade] = \frac{q(q+1)(1-(q-q^2)^{n/2})}{2(1-q-q^2)}$ - "correct" cascade
- $Pr[Down\ cascade] = \frac{(q-2)(q-1)(1-(q-q^2)^{n/2})}{2(1-q-q^2)}$ - "incorrect" cascade



Information cascades

- Cascades start when agents have incomplete information and observe actions of others
- Cascades very easy to start (2 +)
- With large number of people a cascade happens almost surely $\lim_{n\to\infty}(q-q^2)^{n/2} \to 0$
- Cascades prevent information aggregation ("wisdom of crowd"), start based on little information
- Cascades can be wrong - incorrect cascade
- Cascades easy to break (stop)
- Very important early actions/actors in cascades
- Extensions: don't see all the previous decisions, various strength of private signals, different payoff etc

References

- A simple model of herd behavior, A Banerjee, The quarterly journal of economics, vol CVII, Issue 3, pp 797 -817, 1992
- A Theory of Fads, Fashion, Custom, and Cultural Change as Information Cascades, S. Bikhchandani, D Hirshleifer and I.Welch, 1992
- Information Cascades in the Laboratory, L. Anderson and C. Halt
- Following the Herd, Pierre Lemieux