

Diffusion of information and social contagion

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- 1 Rumor spreading models
 - Homogenous approximation
 - Mean field model
 - Examples

- 2 Propagation tree
 - Experimental results

Social contagion

Social contagion phenomena refer to various processes that depend on the individual propensity to adopt and diffuse knowledge, ideas, information.

- Similar to epidemiological models:
 - "susceptible" - an individual who has not learned new information
 - "infected" - the spreader of the information
 - "recovered" - aware of information, but no longer spreading it
- Two main questions:
 - if the rumor reaches high number of individuals
 - rate of infection spread

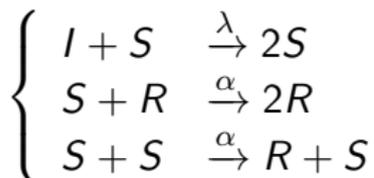
*We will talk about social diffusion, not a physical diffusion process, where conservation laws present

Rumor spreading model

Daley-Kendal model, 1964

Maki -Thomson model, 1973

- Compartmental fully mixed model:
 - "ignorants" - people ignorant to the rumor, I
 - "spreaders" - people who heard and actively spreading rumor, S
 - "stiflers" - heard, but ceased spreading it, R
- Rumor spreads by pair wise contacts
 - λ - rate of ignorant - spreader contacts
 - α - rate of spreader-spreader and spreader-stifler contacts



Rumor spreading model

Homogeneous approximation (lattice or exponential network)

on average $\langle k \rangle$ neighbors

$$i(t) = I(t)/N, s(t) = S(t)/N, r(t) = R(t)/N$$

$$\frac{di(t)}{dt} = -\lambda \langle k \rangle i(t) s(t)$$

$$\frac{ds(t)}{dt} = \lambda \langle k \rangle i(t) s(t) - \alpha \langle k \rangle s(t) [s(t) + r(t)]$$

$$\frac{dr(t)}{dt} = \lambda \langle k \rangle s(t) [s(t) + r(t)]$$

$$i(t) + s(t) + r(t) = 1$$

Rumor spreading model

- time limiting case $r_\infty = \lim_{t \rightarrow \infty} r(t)$:

$$r_\infty = 1 - e^{-(1+\lambda/\alpha)r_\infty}$$

- non-zero solution exists when:

$$\left. \frac{d}{dr_\infty} \left(1 - e^{-(1+\lambda/\alpha)r_\infty} \right) \right|_{r_\infty=0} > 1$$

- threshold:

$$1 + \lambda/\alpha > 1$$

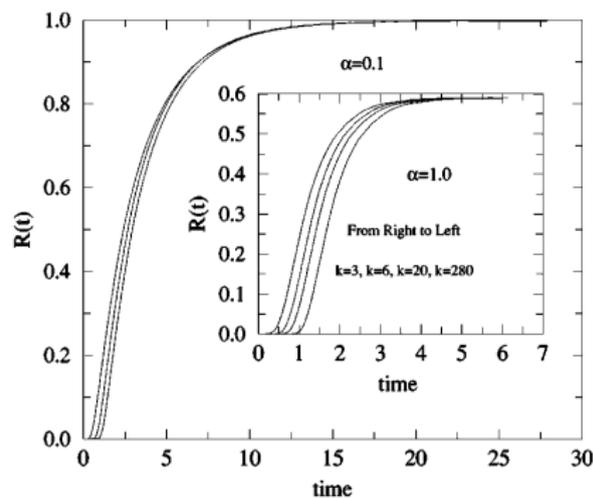
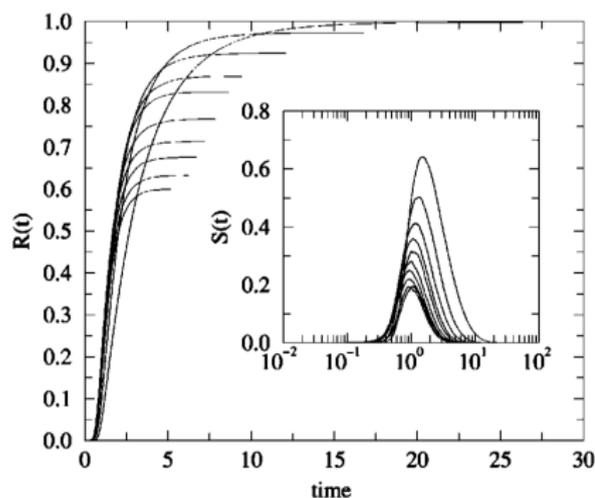
- No rumor epidemic threshold!

Rumor spreading model

Mean-field approach: $i_k(t)$, $s_k(t)$, $r_k(t)$ - densities of ignorants, spreaders and stiflers with connectivity k

$$\begin{aligned}\frac{di_k(t)}{dt} &= -\lambda \langle k \rangle i_k(t) \sum_{k'} \frac{k' P(k') s_{k'}(t)}{\langle k \rangle} \\ \frac{ds_k(t)}{dt} &= \lambda \langle k \rangle i_k(t) \sum_{k'} \frac{k' P(k') s_{k'}(t)}{\langle k \rangle} \\ &\quad - \alpha \langle k \rangle s(t) \sum_{k'} \frac{k' P(k') [s_{k'}(t) + r_k(t)]}{\langle k \rangle} \\ \frac{dr_k(t)}{dt} &= \lambda \langle k \rangle s_k(t) \sum_{k'} \frac{k' P(k') [s_{k'}(t) + r_k(t)]}{\langle k \rangle}\end{aligned}$$

Rumor spreading model



Density of stiflers as function of time (inset - density of spreaders)

Left: as a function of $\alpha = 1.0..0.1$ bottom up

Right: as a function of k

from Moreno et.al, 2004

Rumor spreading model

Rumor dynamics:

- Whenever a spreader contacts an ignorant, the ignorant becomes a spreader at a rate λ .
- When a spreader contacts another spreader or a stifler the initiating spreader becomes a stifler at a rate α .
- Spreader may also cease spreading a rumor spontaneously (without the need for any contact) at a rate δ

Found:

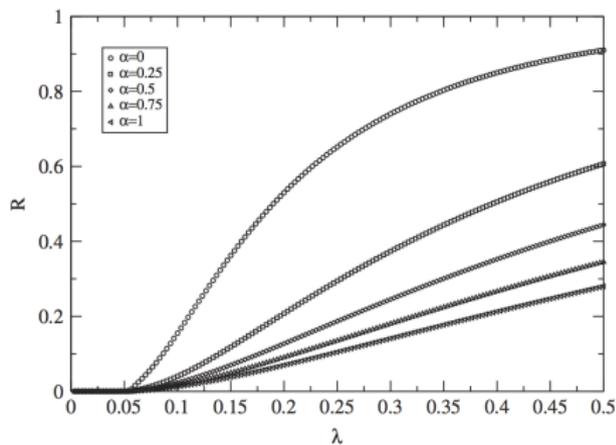
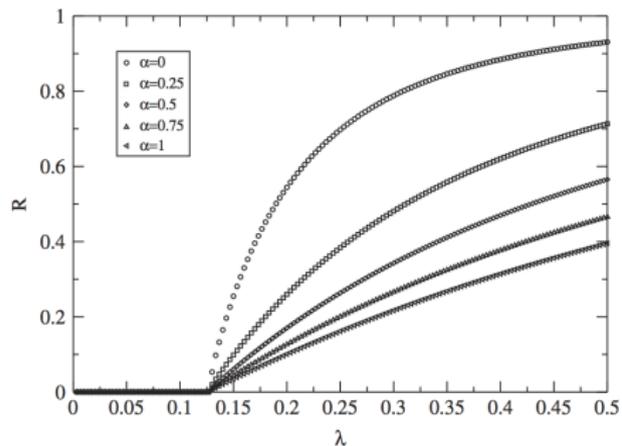
- homogeneous networks and random graphs have epidemic threshold:

$$\frac{\lambda}{\delta} \geq \frac{\langle k \rangle}{\langle k^2 \rangle}$$

- in scale-free networks threshold vanishes as size growth

from Nekovee et.al, 2007

Rumor spreading model



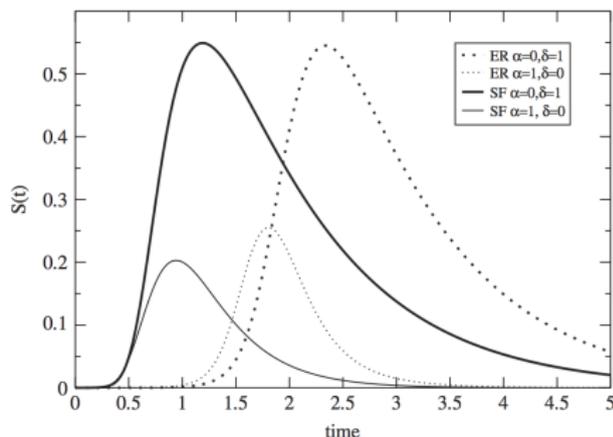
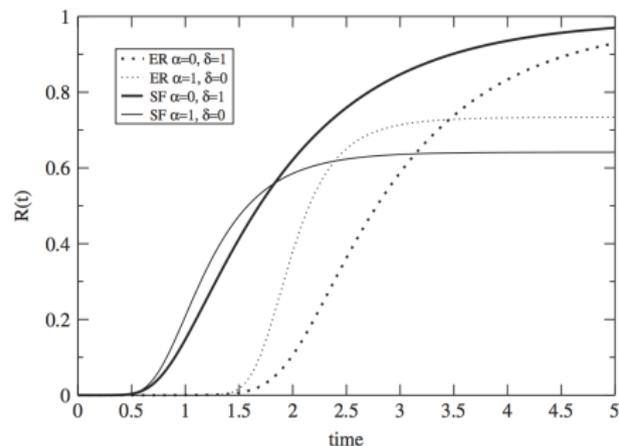
The final rumor size as a function of spreading rate λ .

Left: random graph model, RG (ER)

Right: scale free network, SF

from Nekovee et.al, 2007

Rumor spreading model

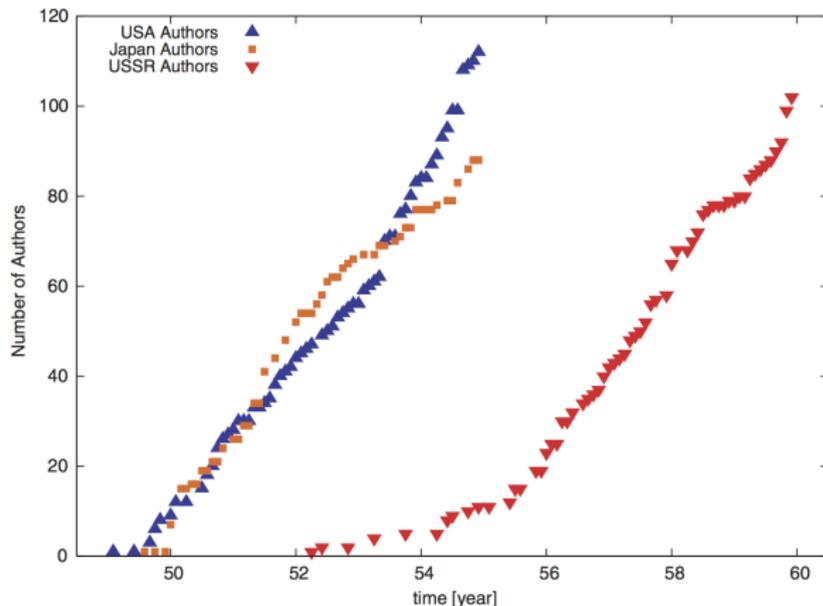


Time evolution of the density of stiflers (left) and spreaders (right) for RG (ER) and SF networks

from Nekovee et.al, 2007

The power of good idea

The power of a good idea: the spread of Feynman diagrams through the theoretical physics community

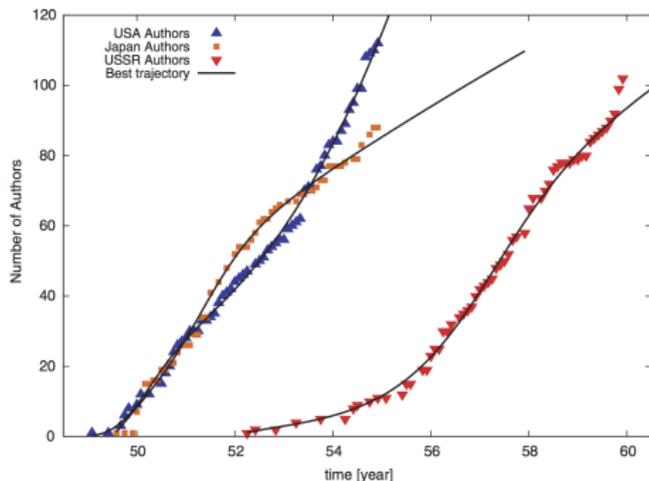


from Luis M.A. Bettencourt et.al, 2005

The power of good idea

Parametric modeling

$$\begin{cases} \dot{S} = \Lambda - \beta S \frac{I}{N} - bS \frac{Z}{N} - \mu S, \\ \dot{E} = (1-p)\beta S \frac{I}{N} + (1-l)bS \frac{Z}{N} - \rho E \frac{I}{N} - \varepsilon E - \mu E, \\ \dot{I} = p\beta S \frac{I}{N} + \rho E \frac{I}{N} + \varepsilon E - \mu I, \\ \dot{Z} = lbS \frac{Z}{N} - \mu Z. \end{cases}$$

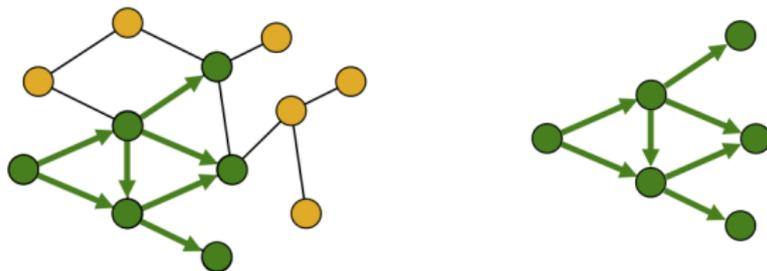


2 types of infected, 9 parameters, 4 initial conditions

from Luis M.A. Bettencourt et.al, 2005

Propagation tree

Rumors/news/infection spreads over the edges: propagation tree (graph)

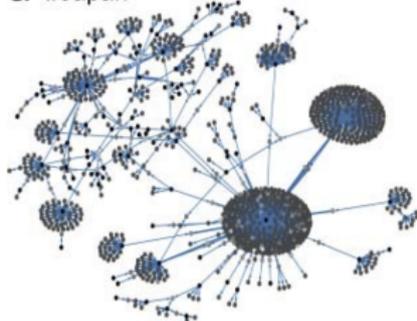


Sometimes called cascade

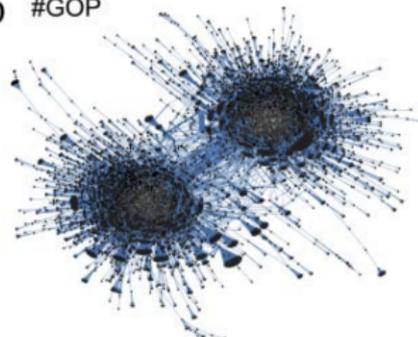
Propagation tree

Mem diffusion on Twitter

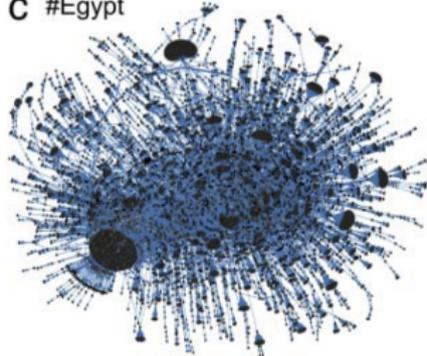
a #Japan



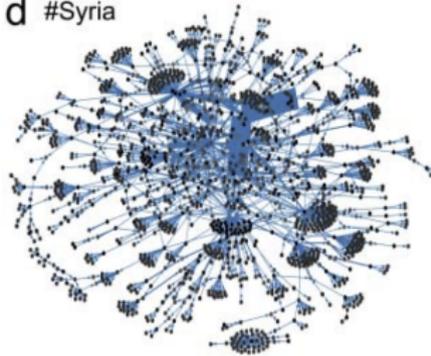
b #GOP



c #Egypt



d #Syria



Branching process

Simple contagion model:

- each infected person meets k new people
- independently transmits infection with probability p

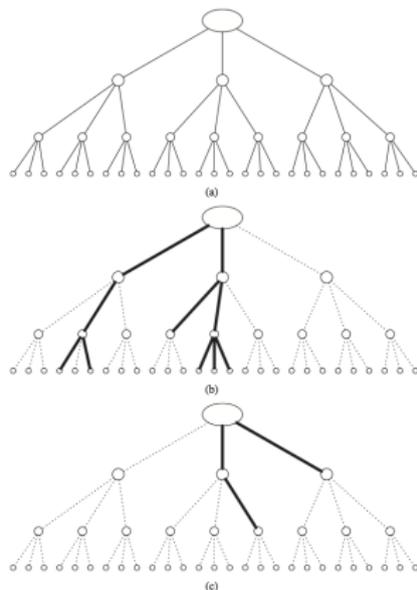
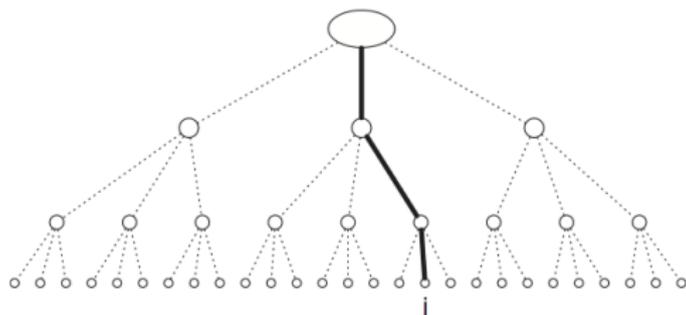


image from David Easley, Jon Kleinberg, 2010

Branching process

Galton-Watson branching random process, $R_0 = pk$

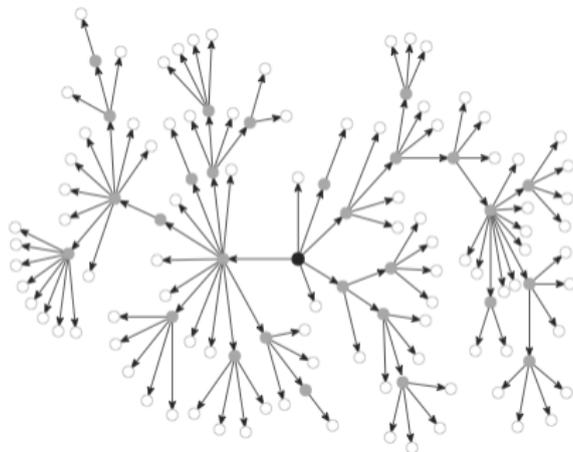
- if $R_0 = 1$, the mean of number of infected nodes does not change
- if $R_0 > 1$, the mean grows geometrically as R_0^n
- if $R_0 < 1$, the mean shrinks geometrically as R_0^n



$R_0 = pk = 1$ - point of phase transition

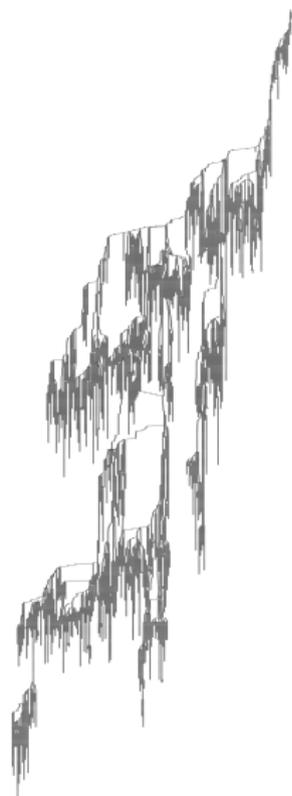
Online newsletter experiment

- Online email newsletter + reward for recommending it
- 7,000 seed, 30,000 total recipients, around 7,000 cascades, size from 2 nodes to 146 nodes, max depth 8 steps
- on average $k = 2.96$, infection probability $p = 0.00879$
- Surprisingly: no loops, triangles, closed paths

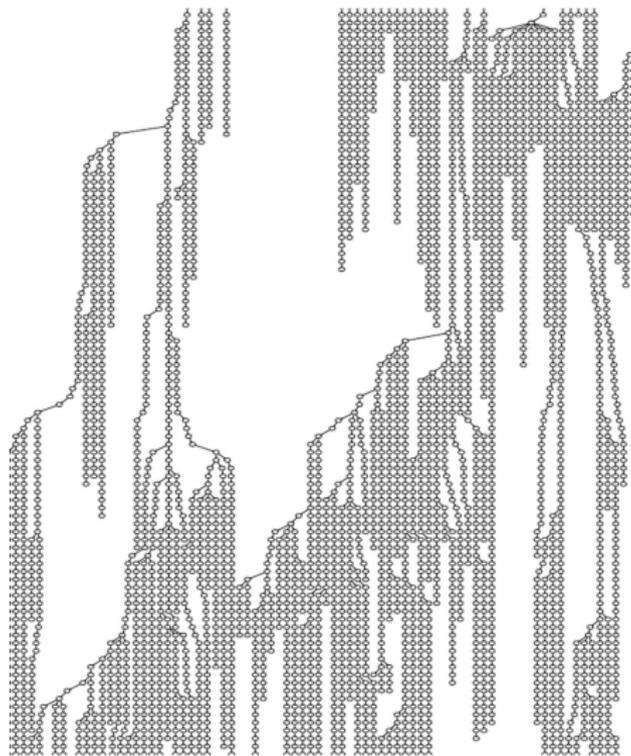


Internet chain letters

- Petitions 2002-2003 opposing war in Iraq: "sign and forward to friends"
- 20,000 distinct signatories in 637 chains
- This tree - 18,119 nodes, 94.3% only one child
- median nodes depth 288, tree width 82

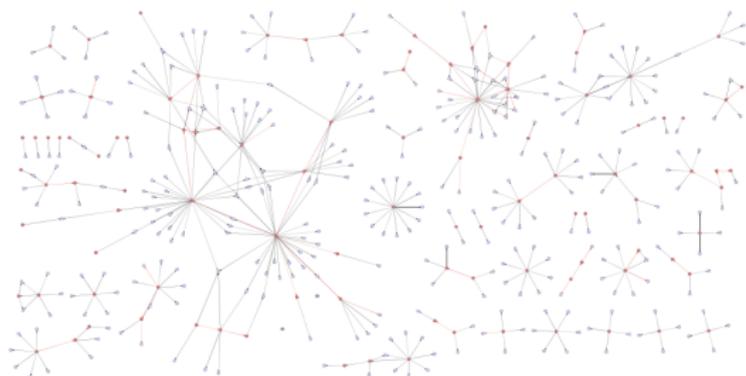


Internet chain letters

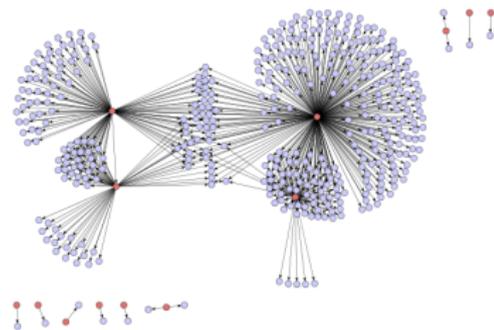


Liben-Nowell, Kleinberg, 2009

Online recommendation network



(a) Medical book



(b) Japanese graphic novel

Large on-line retailer. After purchase one could send recommendation to a friend, both receive discount/referral credit

Leskovec et.al, 2006

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