Spatial Model of Segregation

Leonid E. Zhukov

School of Data Analysis and Artificial Intelligence
Department of Computer Science
National Research University Higher School of Economics

Structural Analysis and Visualization of Networks

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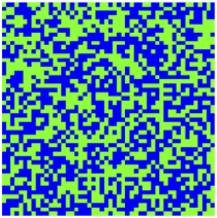
Spatial model of segregation

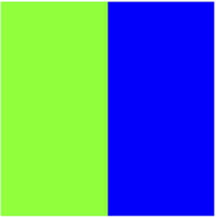
"Dynamic Models of Segregation", Thomas Schelling, 1971

- Micro-motives and macro-behavior
- Personal preferences lead to collective actions
- Global patterns of spatial segregation from homophily at a local level
- Segregated race, ethnicity, native language, income
- Cities are strongly racially segregated. Are people that racists?
- Agent based modeling: agents, rules (dynamics), aggregation

Segregation



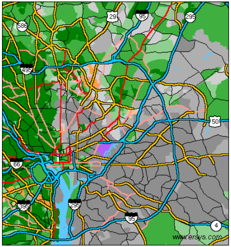
Integrated pattern



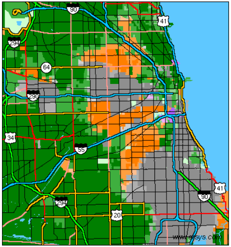
Segregated pattern

Racial segregation

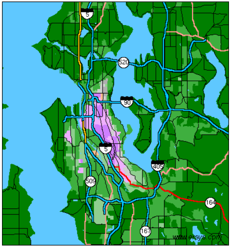
New York



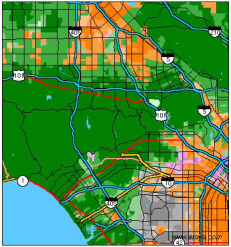
Washington



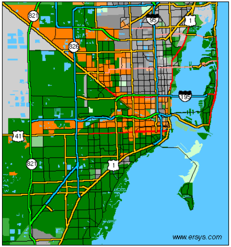
Chicago



Seattle

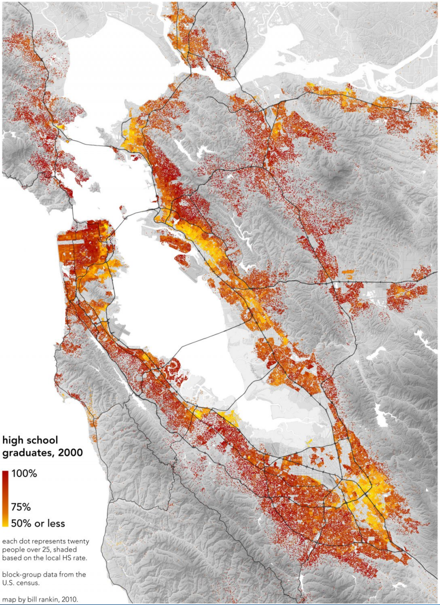


Los Angeles

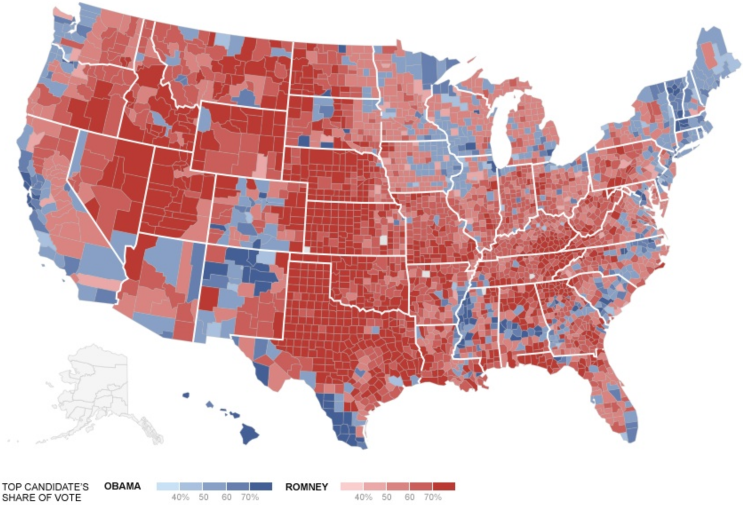


Miami

Bay area high school graduates



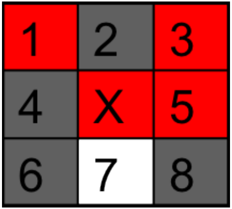
2012 US Presidential Elections Map



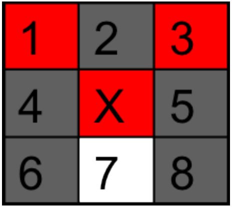
Schelling's model of segregation

- Population consists of 2 types of agents
- Agent reside in the cells of the grid (2-dimensional geography of a city), 8 neighbors
- Some cells contain agents, some unpopulated
- Every agent wants to have at least some fraction of agents (threshold) of his type as neighbor (satisfied agent)
- On every round every unsatisfied agent moves to a satisfactory empty cell.
- Continues until everyone is satisfied or can't move

Spatial segregation



satisfied agent



unsatisfied agent

- preference threshold $\lambda = 3/7$

Spatial segregation

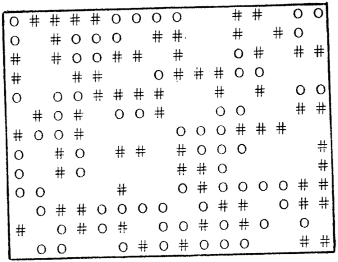


Fig.7

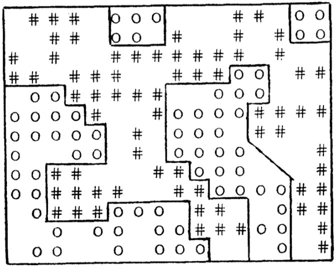
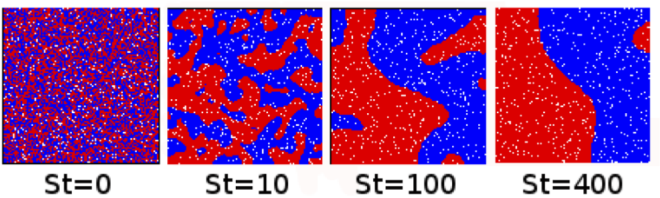


Fig.10

T. Schelling, 1971

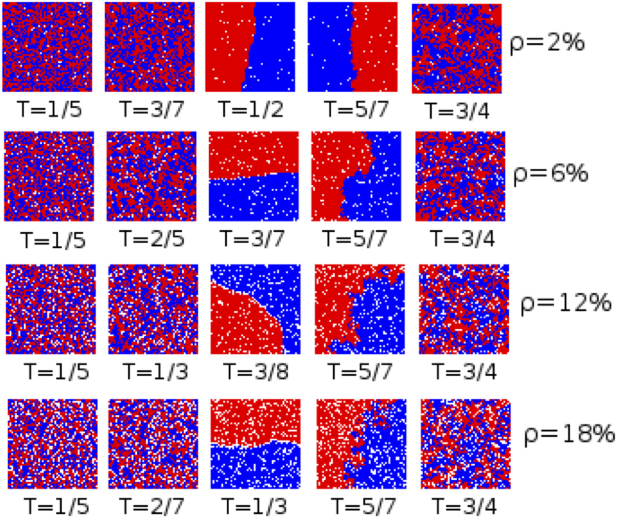
Spatial segregation

vacancy 5%, tolerance $\lambda = 0.5$



L. Gauvin et.al. 2009

Spatial segregation



L. Gauvin et.al. 2009

Model

- N - nodes, θ - fraction of occupied by A and B

$$n_A + n_B = \theta \cdot N$$

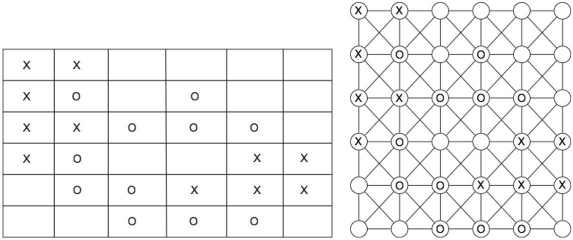
- Proportion of "unlike" nearest neighbors, $k_i = \#NN$

$$P_i = \begin{cases} \#n_B/k_i, \text{if } \ i \in A \\ \#n_A/k_i, \text{if } \ i \in B \end{cases}$$

- Utility function, λ - sensitivity (tolerance threshold) level

$$u_i = \begin{cases} 1, \text{if } \ P_i \leq \lambda \\ 0, \text{if } \ P_i > \lambda \end{cases}$$

- Every node moves to maximize its utility



(a) (b)

Algorithm

- time steps $1..T$
- At every time step randomly select an agent, compute utility
- If utility is $u = 0$ move to an empty location to maximize utility
- Movements: 1) random location 2) nearest available location
- Repeat until either all utilities are maximized $\sum_i u_i = \theta N$ or reaches "frozen" state, no place to move, then $\sum_i u_i < \theta N$
- Total utility of society

$$U = \sum_i u_i$$

Measuring segregation

- Schilling's solid mixing index

$$M = \frac{1}{n_A + n_B} \sum_i P_i$$

- Freeman's segregation index

$$F = 1 - \frac{e^*}{E(e^*)}$$

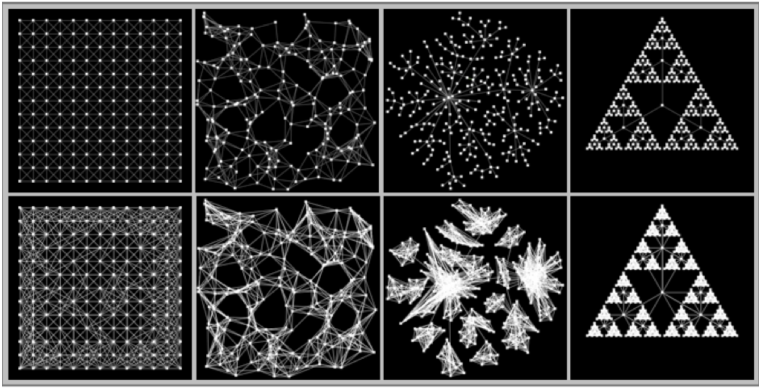
$e^* = \frac{e_{AB}}{(e_{AB}+e_{AA}+e_{BB})}$ - observed proportion of between group ties,
$E(e^*) = \frac{2n_A n_B}{(n_A+n_B)(n_A+n_B-1)}$ - expected proportion for random ties

- Assortative mixing

$$Q = \frac{1}{2m} \sum_{ij} (A_{ij} - \frac{k_i k_j}{2m}) \delta(c_i, c_j)$$

Spatial segregation on networks

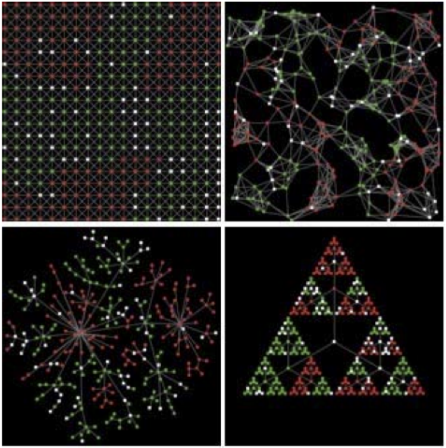
Fixed degree $k = 10$ neighboring graphs: regular, random, scale-free, fractal



Arnaud Banos, 2010

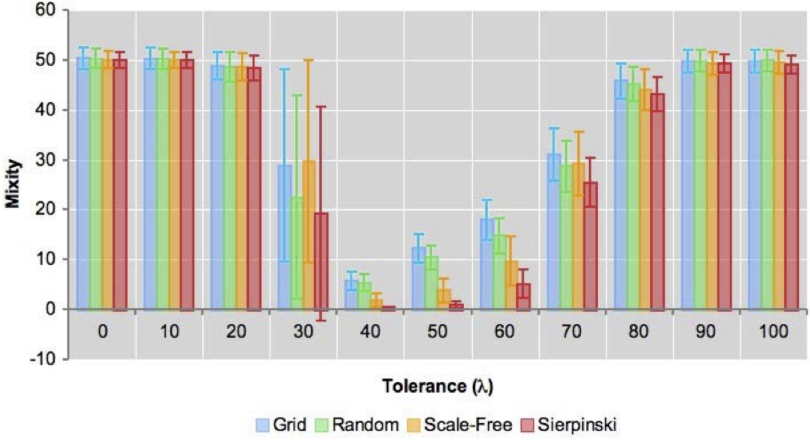
Spatial segregation on networks

$\lambda = 0.5, \theta = 0.8$



Banos, 2010

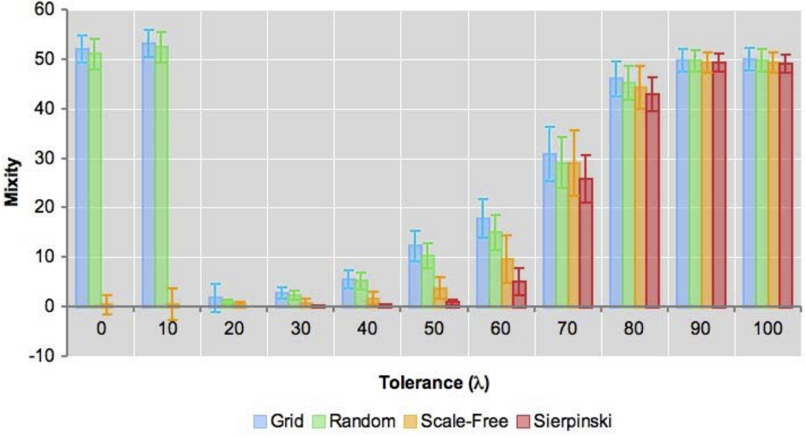
Spatial segregation on networks



Banos, 2010

Spatial segregation on networks

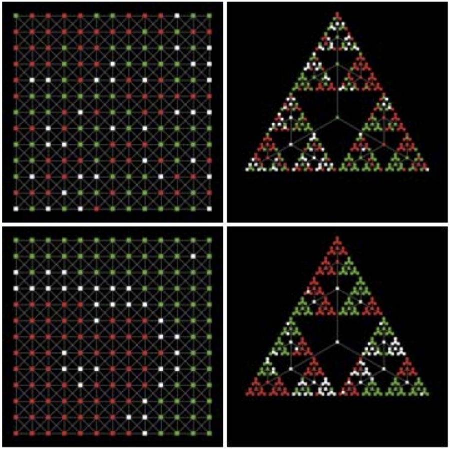
$\nu = 10\%$ of random "noise" added for decision to avoid freezes



Banos, 2010

Spatial segregation on networks

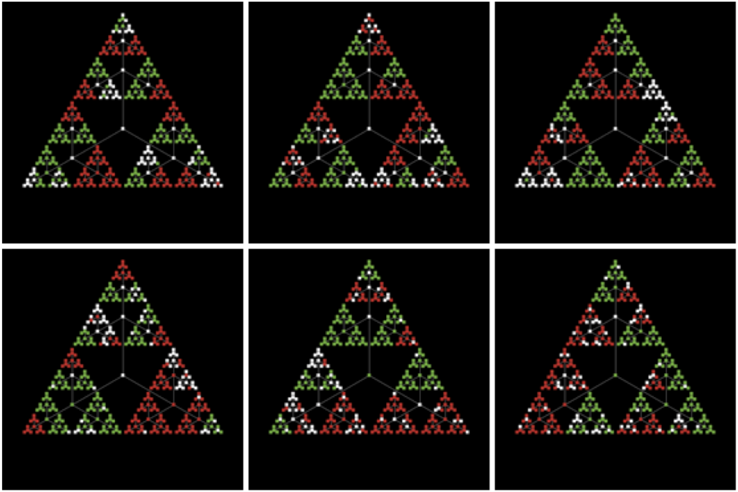
$\lambda = 0.3, \theta = 0.8$ Sensitivity to initial conditions



Banos, 2010

Spatial segregation on networks

$\nu = 0.1, \theta = 0.8, \lambda = 0..0.5$



Banos, 2010

Mathematical description

- Agents of two types A, B
- $\sigma_i = \pm 1$ if site i is occupied, $\sigma_i = 0$ if empty
- System state vector: $\sigma = (\sigma_1 ... \sigma_N)$
- Adjacency matrix A_{ij}
- Fraction of vacant sites $\theta = 1/N \sum_i (1 - |\sigma_i|)$
- Proportion of "unlike" neighbours

$$P_i = \frac{\sum_j A_{ij}(|\sigma_i \sigma_j| - \sigma_i \sigma_j)}{\sum_j A_{ij} |\sigma_i \sigma_j|}$$

Summary

- Spatial segregation is taking place even though no individual agent is actively seeking it (minor preferences, high tolerance)
- Network structure does affect segregation
- Fixed characteristics (race) can become correlated with mutable (location)

References

- Dynamic Models of Segregation, Thomas C. Schelling, 1971
- Segregation in Social Networks, Linton Freeman, 1978
- Gauvin L, Vannimenus J, Nadal JP. Phase diagram of a Schelling segregation model. The European Physical Journal B, 70:293-304, 2009
- Arnaud Banos. Network effects in Schelling's model of segregation: new evidences from agent-based simulations. 2010