Spatial Model of Segregation

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Structural Analysis and Visualization of Networks
"Dynamic Models of Segregation", Thomas Schelling, 1971

- Micro-motives and macro-behavior
- Personal preferences lead to collective actions
- Global patterns of spatial segregation from homophily at a local level
- Segregated race, ethnicity, native language, income
- Cities are strongly racially segregated. Are people that racists?
- Agent based modeling: agents, rules (dynamics), aggregation
Segregation

Integrated pattern  Segregated pattern
Racial segregation

New York

Washington

Chicago

Seattle

Los Angeles

Miami
Bay area high school graduates

Each dot represents twenty people over 25, shaded based on the local HS rate. Block group data from the U.S. census.

Map by Bill Rankin, 2010.
Population consists of 2 types of agents

Agent reside in the cells of the grid (2-dimensional geography of a city), 8 neighbors

Some cells contain agents, some unpopulated

Every agent wants to have at least some fraction of agents (threshold) of his type as neighbor (satisfied agent)

On every round every unsatisfied agent moves to a satisfactory empty cell.

Continues until everyone is satisfied or can’t move
Spatial segregation

- preference threshold $\lambda = \frac{3}{7}$

Diagram showing two scenarios:
- Satisfied agent: Positions marked with red squares
- Unsatisfied agent: Positions marked with red squares
Spatial segregation

T. Schelling, 1971

Fig. 7

Fig. 10
Spatial segregation

vacancy 5%, tolerance $\lambda = 0.5$

L. Gauvin et al. 2009
Spatial segregation

L. Gauvin et al. 2009
Model

- \( N \) - nodes, \( \theta \) - fraction of occupied by \( A \) and \( B \)

\[
n_A + n_B = \theta \cdot N
\]

- Proportion of "unlike" nearest neighbors, \( k_i = \#NN \)

\[
P_i = \begin{cases} 
\#n_B/k_i, & \text{if } i \in A \\
\#n_A/k_i, & \text{if } i \in B
\end{cases}
\]

- Utility function, \( \lambda \) - sensitivity (tolerance threshold) level

\[
u_i = \begin{cases} 
1, & \text{if } P_i \leq \lambda \\
0, & \text{if } P_i > \lambda
\end{cases}
\]

- Every node moves to maximize its utility
Spatial segregation

(a) 

(b)
Algorithm

- time steps 1.. $T$
- At every time step randomly select an agent, compute utility
- If utility is $u = 0$ move to an empty location to maximize utility
- Movements: 1) random location 2) nearest available location
- Repeat until either all utilities are maximized $\sum_i u_i = \theta N$
  or reaches ”frozen” state, no place to move, then $\sum_i u_i < \theta N$
- Total utility of society

$$U = \sum_i u_i$$
Measuring segregation

- Schilling’s solid mixing index
  \[ M = \frac{1}{n_A + n_B} \sum_i P_i \]

- Freeman’s segregation index
  \[ F = 1 - \frac{e^*}{E(e^*)} \]
  \[ e^* = \frac{e_{AB}}{(e_{AB} + e_{AA} + e_{BB})} \] - observed proportion of between group ties,
  \[ E(e^*) = \frac{2n_An_B}{(n_A+n_B)(n_A+n_B-1)} \] - expected proportion for random ties

- Assortative mixing
  \[ Q = \frac{1}{2m} \sum_{ij} (A_{ij} - \frac{k_i k_j}{2m}) \delta(c_i, c_j) \]
Spatial segregation on networks

Fixed degree $k = 10$ neighboring graphs: regular, random, scale-free, fractal

Arnaud Banos, 2010
Spatial segregation on networks

$\lambda = 0.5, \theta = 0.8$

Banos, 2010
Spatial segregation on networks

![Graph showing mixity vs. tolerance for different network structures.]

Banos, 2010
Spatial segregation on networks

\[ \nu = 10\% \text{ of random "noise" added for decision to avoid freezes} \]

Banos, 2010
Spatial segregation on networks

$\lambda = 0.3, \theta = 0.8$ Sensitivity to initial conditions

Banos, 2010
Spatial segregation on networks

\[ \nu = 0.1, \theta = 0.8, \lambda = 0.05 \]

Banos, 2010
Mathematical description

- Agents of two types A, B
- \( \sigma_i = \pm 1 \) if site \( i \) is occupied, \( \sigma_i = 0 \) if empty
- System state vector: \( \sigma = (\sigma_1...\sigma_N) \)
- Adjacency matrix \( A_{ij} \)
- Fraction of vacant sites \( \theta = 1/N \sum_i (1 - |\sigma_i|) \)
- Proportion of "unlike" neighbours

\[
P_i = \frac{\sum_j A_{ij}(|\sigma_i\sigma_j| - \sigma_i\sigma_j)}{\sum_j A_{ij}|\sigma_i\sigma_j|}
\]
Spatial segregation is taking place even though no individual agent is actively seeking it (minor preferences, high tolerance)

Network structure does affect segregation

Fixed characteristics (race) can become correlated with mutable (location)
- Dynamic Models of Segregation, Thomas C. Schelling, 1971
- Segregation in Social Networks, Linton Freeman, 1978
- Arnaud Banos. Network effects in Schelling’s model of segregation: new evidences from agent-based simulations. 2010