Strategic Network Formation

Leonid E. Zhukov

School of Data Analysis and Artificial Intelligence
Department of Computer Science
National Research University Higher School of Economics

Structural Analysis and Visualization of Networks

Lecture outline

1. Strategic network formation
   - Connections model
   - Co-author model
2. Network pairwise stability
3. Network efficiency

Strategic network formation

M. Jackson, A. Wolinksy, 1996
"A Strategic Model of Social and Economic Networks"

- why networks becomes the way they are
- people (agents) making rational choices establishing connections
- maximizing individual utility (incentives)
- connections brings costs and benefits
- stability of the network
- social efficiency (best for the society)
- friendship, professional, political, trade networks

Modeling choices

Agent based modeling:

- Payoff (benefit) and costs of forming links
- Pairwise connections - mutual agreement (directed/undirected)
- Individual or coordinated changes in network structure
- Intensity of the connections
- "Rules" for connections / decision making
- Possibility of errors

Connections model

- $u_i(G)$ - payoff to agent i in the network G
- Distance-based utility function

$$u_i(G) = \sum_j \delta_{ij}^{l_{ij}} - \sum_{j \in \mathcal{N}_i} c_{ij}$$

l_{ij} - shortest path, δ_{ij} - benefit value, c_{ij} - cost of link i to j, $\delta < 1$, $c > 0$

- Symmetric version

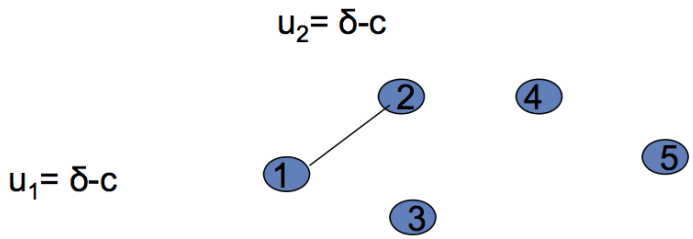
$$u_i(G) = \sum_j \delta^{l_{ij}} - d_i \cdot c$$

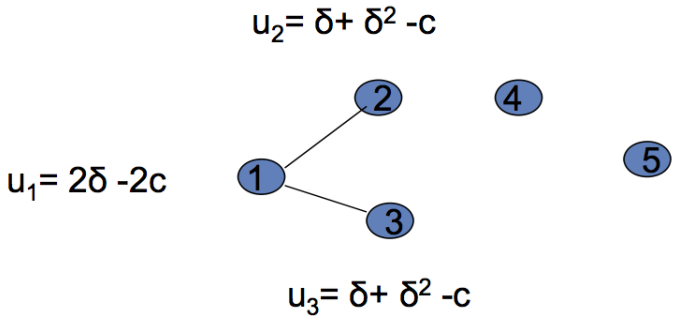
d_i - node degree,

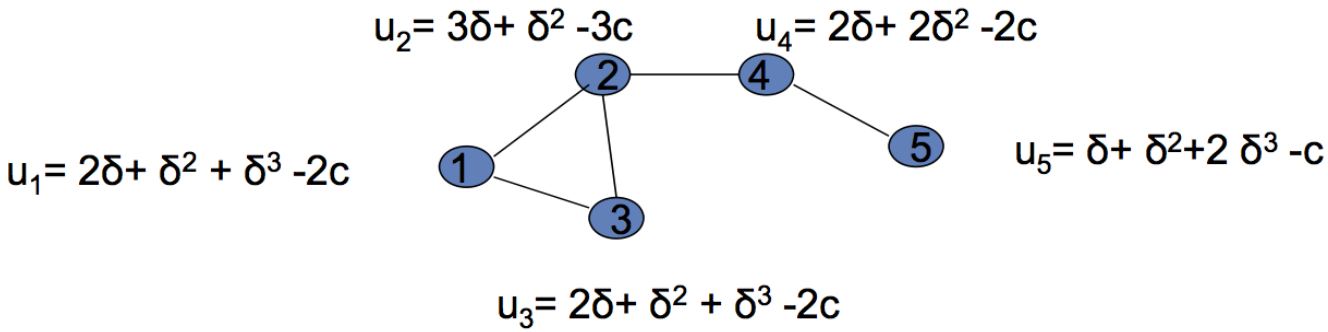
- Positive externalities - positive impact from others forming relationships

Jackson, A. Wolinksy, 1996

Distance-based utility function







Co-author model

- Utility function:

$$u_i(G) = \sum_{j \in N_i} \left(\frac{1}{d_i} + \frac{1}{d_j} + \frac{1}{d_i d_j}\right) = 1 + \sum_{j \in N_i} \left(\frac{1}{d_j} + \frac{1}{d_i d_j}\right), \quad d_i \neq 0$$

  $u_i(G) = 1$ if $d_i = 0$, d_i - node degree

- Negative externalities - negative impact from others forming relationships

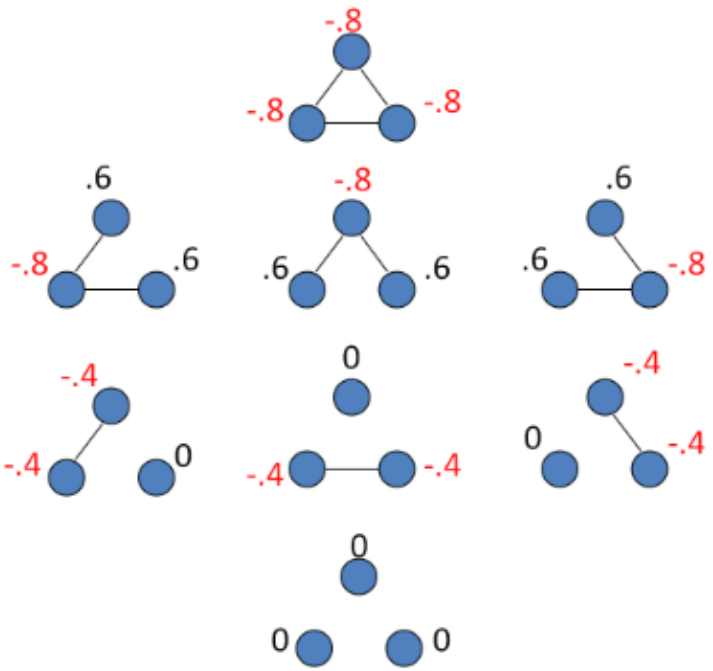
Pairwise stability

- Evolution: forming a link - mutual consent, removing a link - one person decision
- Network is pairwise stable if no agent wants to remove a link and no two players want to add a link
- Pairwise stability of network G:
  1) No agent gains by removing a link
  2) No **two** agents both gain from adding a link

$$\begin{aligned} \forall i \ u_i(G) &\geq u_i(G - e_{ij}) \\ \forall i,j \text{ if } u_i(G + e_{ij}) &> u_i(G), \text{then } u_j(G + e_{ij}) < u_j(G) \end{aligned}$$

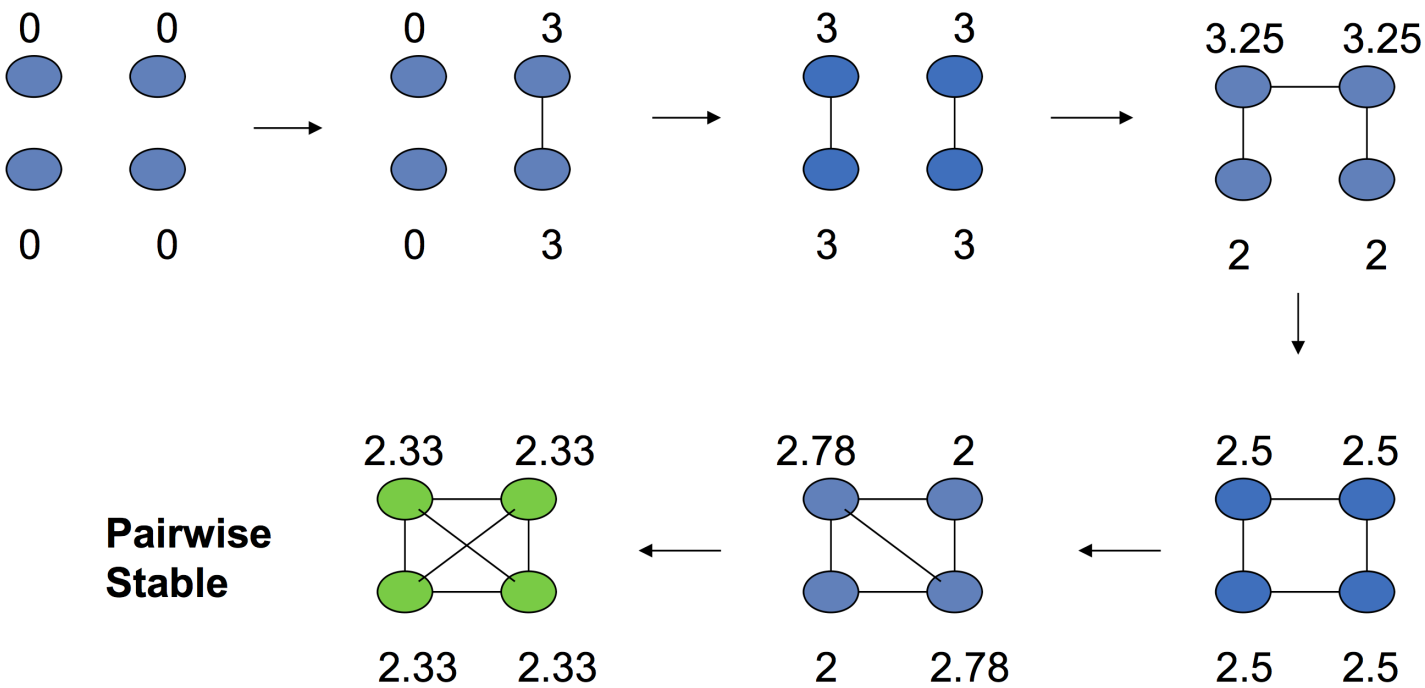
- Weak concept
  1) considers removing one link at a time
  2) considers only forming one pair at a time

Network stability



$\delta = 0.999,\ c = 1.4$

Network stability



Network efficiency

- Strong efficiency - "best network", maximize total utility for the society

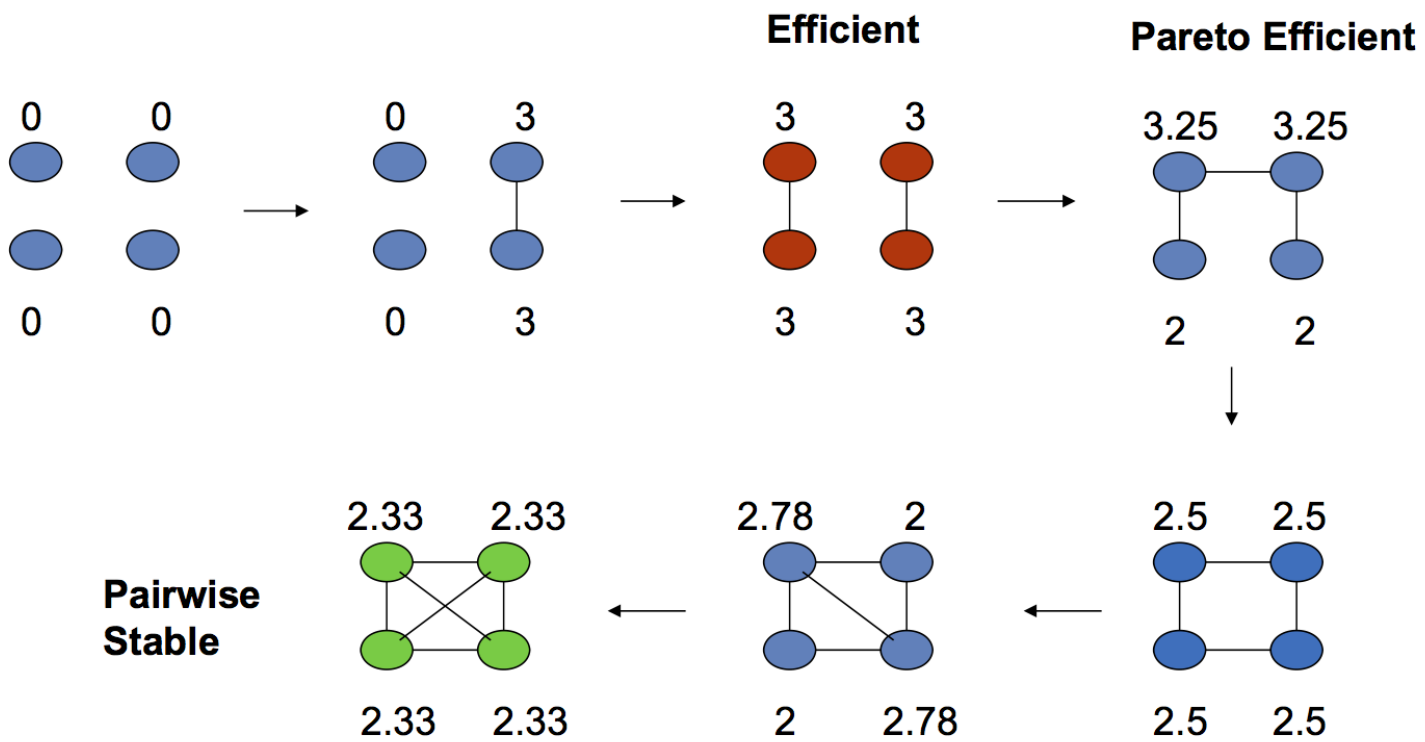
$$G^* = \max_G \sum_i u_i(G)$$

- Pareto efficiency - no other network where everybody not worse, some better

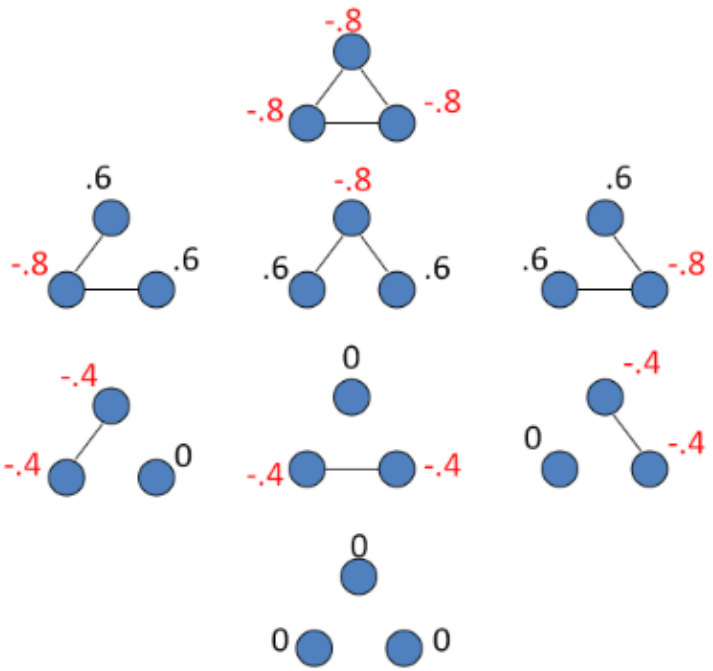
$$\begin{aligned} \nexists\, G' : \; u_i(G') &\geq u_i(G) \text{ for all } i \\ \text{and } u_i(G') &> u_i(G) \text{ for one } i \end{aligned}$$

- Efficiency $\Rightarrow$ Pareto efficiency

Network Efficiency



Network efficiency



$\delta = 0.999,\ c = 1.4$

Efficient and pairwise stable networks

Symmetric connections model:

- Low connections cost: complete network is efficient and stable

$$c < \delta - \delta^2$$

- Medium connections cost: star network is efficient and stable (only when $c < \delta$)

$$\delta - \delta^2 < c < \delta + (n-2)\delta^2/2$$

- High connections cost: empty network is efficient and stable

$$c > \delta(n-2)\delta^2/2$$

Jackson, Wolinksy, 1996

References

- A Strategic Model of Social and Economic Networks, M. Jackson, A. Wolinksy, J. of Economic Theory, 71, pp44-74, 1996.
- The Economics of Social Networks. California Institute of Technology, 2005.

Module summary

1. Diffusion on network
2. Epidemics
3. Epidemics on networks
4. Social contagion and information spread
5. Diffusion of innovation and influence maximization
6. Social learning (DeGroot model)
7. Label propagation
8. Link prediction
9. Spatial segregation
10. Strategic network formation