



NATIONAL RESEARCH  
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# Diffusion of Innovation

Social Network Analysis. MAGoLEGO course.

## Lecture 7

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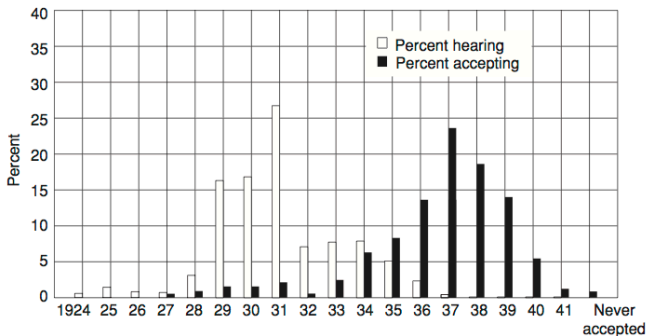
Propagation process:

- Information based models:
  - ideas, knowledge
  - virus and infection
  - rumors, news
- Decision based models:
  - adoption of innovation
  - joining political protest
  - purchase decision

Local individual decision rules will lead to very different global results.

"microscopic" changes → "macroscopic" results

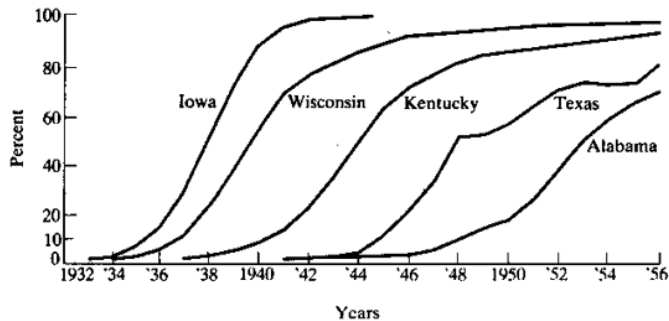
## Ryan-Gross study of hybrid seed corn delayed adoption (after first exposure)



## Information effect vs adopting of innovation

Ryan and Gross, 1943

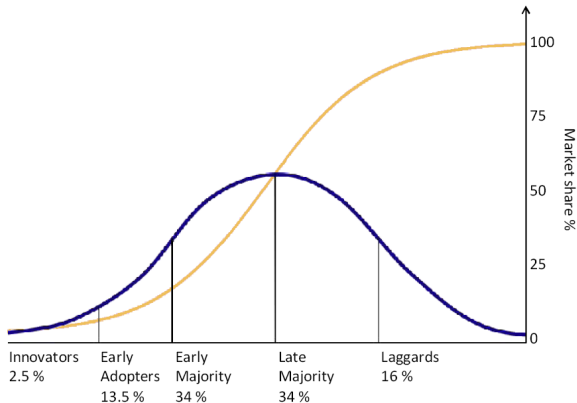
## Hybrid corn adoption



## Percentage of total acreage planted

Griliches, 1957

Everett Rogers, "Diffusion of innovation" book, 1962



Frank Bass, 1969, "A new product growth model for consumer durables"

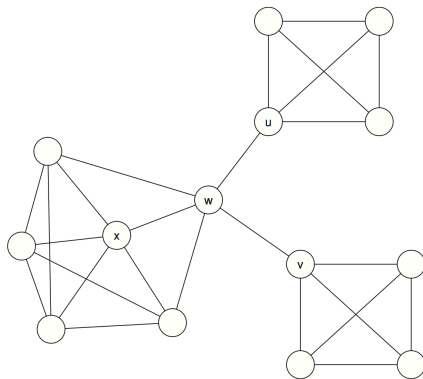
What influences potential adopters:

- relative advantage of the innovation
- compatibility with current ways of doing things
- complexity of the innovation
- triability - the ease of testing
- observability of results

Some questions remain:

- how a new technology can take over?
- who different technologies coexist?
- what stops new technology propagation?

## From the population level to local structure



Local interaction game: Let  $u$  and  $v$  are players, and  $A$  and  $B$  are possible strategies

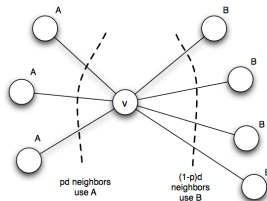
Payoffs

- if  $u$  and  $v$  both adopt behavior  $A$ , each get payoff  $a > 0$
- if  $u$  and  $v$  both adopt behavior  $B$ , each get payoff  $b > 0$
- if  $u$  and  $v$  adopt opposite behavior, each get payoff  $0$

		$w$	
		$A$	$B$
$v$	$A$	$a, a$	$0, 0$
	$B$	$0, 0$	$b, b$



## Network coordination game, direct-benefit effect



		$w$	
		$A$	$B$
$v$	$A$	$a, a$	$0, 0$
	$B$	$0, 0$	$b, b$

Node  $v$  to make decision  $A$  or  $B$ ,  $p$  - portion of type  $A$  neighbors to accept  $A$ :

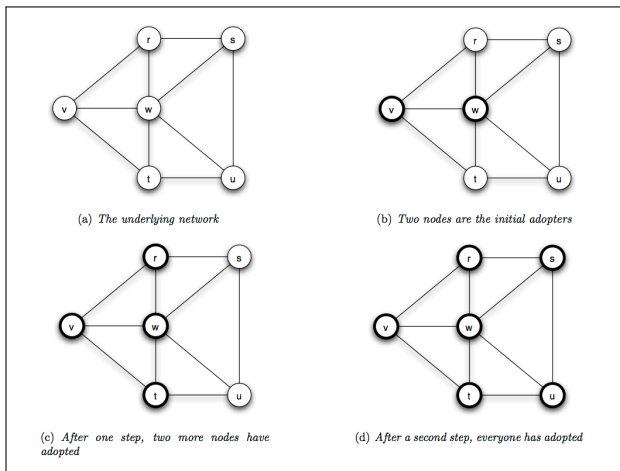
$$a \cdot p \cdot d > b \cdot (1 - p) \cdot d$$

$$p \geq b / (a + b)$$

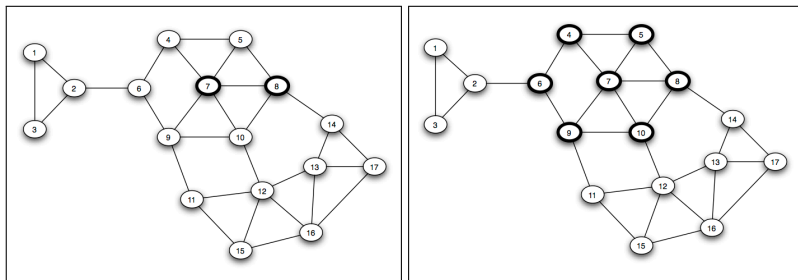
Threshold:

$$q = \frac{b}{a + b}$$

Cascade - sequence of changes of behavior, "chain reaction"

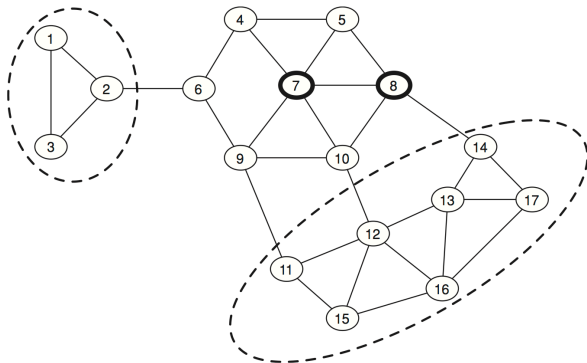


Let  $a = 3$ ,  $b = 2$ , threshold  $q = 2/(2 + 3) = 2/5$



- Let  $a = 3, b = 2$ , threshold  $q = 2/(2 + 3) = 2/5$
- Start from nodes 7,8:  $1/3 < 2/5 < 1/2 < 2/3$
- Cascade size - number of nodes that changed the behavior
- Complete cascade when every node changes the behavior

Group of nodes form a cluster of density  $\rho$  if every node in the set has at least fraction  $\rho$  of its neighbors in the set

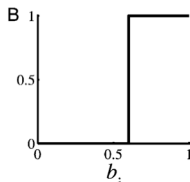


Both clusters of density  $\rho = 2/3$ . For cascade to get into cluster  $q \leq 1 - \rho$ .

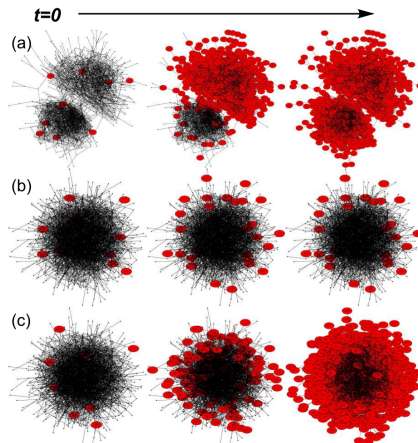
- Influence comes only from NN  $N(i)$  nodes,  $w_{ij}$  influence  $i \rightarrow j$
- Require  $\sum_{j \in N(i)} w_{ji} \leq 1$
- Each node has a random acceptance threshold from  $\theta_i \in [0, 1]$
- Activation: fraction of active nodes exceeds threshold

$$\sum_{\text{active } j \in N(i)} w_{ji} > \theta_i$$

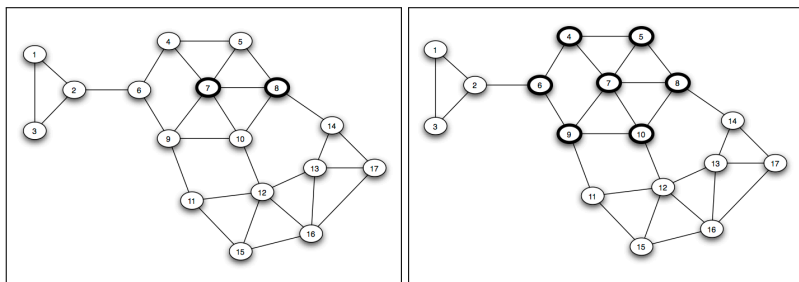
- Initial set of active nodes  $A_0$ , iterative process with discrete time steps
- Progressive process, only nonactive  $\rightarrow$  *active*



multiple seed nodes



(a) Empirical network; (b), (c) - randomized network



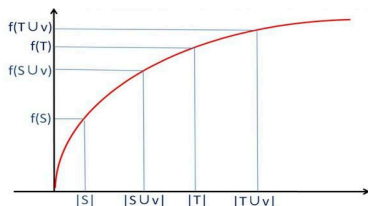
- Initial set of active nodes  $A_0$
- Cascade size  $\sigma(A_0)$  - expected number of active nodes when propagation stops
- Find  $k$ -set of nodes  $A_0$  that produces maximal cascade  $\sigma(A_0)$
- $k$ -set of "maximum influence" nodes
- NP-hard

Greedy maximization algorithm:

Given: Graph and set size  $k$

Output: Maximum influence set  $A$

1. Select a node  $v_1$  that maximizes the influence  $\sigma(v_1)$
2. Fix  $v_1$  and find  $v_2$  such that maximizes  $\sigma(v_1, v_2)$
3. Repeat  $k$  times
4. Output maximum influence set:  $A = \{v_1, v_2 \dots v_k\}$





## Approximation algorithm

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**Algorithm:** Greedy optimization

**Input:** Graph  $G(V, E)$ ,  $k$

**Output:** Maximum influence set  $S$

Set  $S \leftarrow \emptyset$

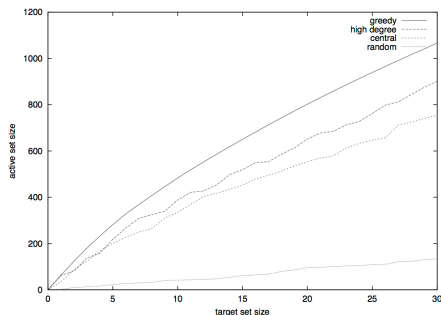
**for**  $i = 1 : k$  **do**

    select  $v = \arg \max_{u \in V \setminus S} (\sigma(S \cup \{u\}) - \sigma(S))$   
     $S \leftarrow S \cup \{v\}$

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## Linear threshold model

network: collaboration graph 10,000 nodes, 53,000 edges



Greedy algorithm finds a set  $S$  such that its influence set  $\sigma(S)$  is  $\sigma(S) \geq (1 - \frac{1}{e})\sigma(S^*)$  from the true optimal (maximal) set  $\sigma(S^*)$

- Contagion, S. Morris, Review of Economic Studies, 67, p 57-78, 2000
- Maximizing the Spread of Influence through a Social Network, D. Kempe, J. Kleinberg, E. Tardos, 2003
- Influential Nodes in a Diffusion Model for Social Networks, D. Kempe, J. Kleinberg, E. Tardos, 2005
- A Simple Model of Global Cascades on Random Networks. D. Watts, 2002.